# HOMEWORK PROBLEMS

## 1. Basic Geometry and Trigonometry

(1) A box has a base which is 2 feet wide and 3 feet long. If the box is 4 feet tall and is open at the top, what is the surface area of the box in square feet?
(2) What is the exact value of the sine of 240 degrees?
(3) A boat sails 1 mile at an angle of 60 degrees from shore. How far is the boat from the shore? Assume that the shore is straight.
(4) What is the sine, cosine and tangent of 300 degrees?
(5) Assume $\theta$ is an angle in the first quadrant. If $\sin\theta = 3/4$, what is $\tan\theta$?

## 2. Chapter 1.1: Functions

(1) Quick Check: 1-5
(2) Exercise Set: 1, 3, 4, 9, 11(a,b), 25

## 3. Chapter 1.3: New functions from old

(1) Quick Check: 1-4
(2) Exercise Set: 29, 30, 33, 35, 37, 41

## 4. Chapter 1.4: Families of Functions

(1) Quick Check: 1-4
(2) Exersice Set: 1-5, 11, 15, 17, 19, 27.

**Solution to 1.4 number 5:**

We need to determine which lines $y = mx + b$ are tangent to the circle $x^2 + y^2 = 9$. First, there are two vertical lines and two horizontal lines. These lines are given by the equations $x = \pm 3$ or $y = \pm 3$. For all other lines, $m \neq 0$ and $|b| > 3$(Justify this for yourself!). So, for each $-1 < x < 1$, there are two tangent lines such that the point of tangency has $x$ as its first coordinate. For example, if $x = \sqrt{3}/2$, the lines $y = x - 3\sqrt{2}$ and $y = -x + 3\sqrt{2}$ are the two tangent lines passing through the points $(3/\sqrt{2}, -3/\sqrt{2})$ and $(3/\sqrt{2}, 3/\sqrt{2})$, respectively.

For other values of $-1 < x < 1$, we need to find a unique solution to

$$x^2 + (mx+b)^3 = 9 \to (m^2+1)x^2 + 2bmx + (b^2-9) = 0.$$

From the quadratic formula, this means that we must have

$$(2bm)^2 - 4(m^2+1)(b^2-9) = 0.$$

If this is true, the unique solution to

$$(m^2+1)x^2 + 2bmx + (b^2-9) = 0$$

is

$$x = \tfrac{-2bm}{2(m^2+1)} = \tfrac{-bm}{m^2+1}.$$

If we plug this value into the equation

$$(m^2+1)x^2 + 2bmx + (b^2-9) = 0$$

we get the equation

$$b^2 = 9(m^2+1).$$

Therefore, other than the vertical and horizontal lines mentioned above, the lines $y = mx + b$ are given by

$$y = mx \pm 3\sqrt{m^2+1}.$$

Notice that this is consistent with the examples above. For the line $y = -x + 3\sqrt{2}$, $m = -1$ and $b = 3\sqrt{2}$.

## 5. Chapter 1.5: Inverse Functions

(1) Quick Check: 1, 2, 4(a)-(d)
(2) Exercise Set: 1, 3, 9, 11, 16, 19, 21, 24, 29, 35, 41(a)-(b)

## 6. Chapter 1.6: Exponentials and Logarithms

(1) Quick Check: 1-5
(2) Exercise Set: 1, 2, 5, 6, 9-33, 51, 52

## 7. Chapter 2.1: Limits

(1) Quick Check: 1-3
(2) Exercise Set: 1-7, 19

## 8. Chapter 2.2: Computing Limits

(1) Quick Check: 1-4
(2) Exercise Set: 3-11(odd), 15-25(odd), 31, 37

## 9. Chapter 2.3: End Behavior

(1) Quick Check: 1-4
(2) Exercise Set: 1-6, 7-27(odd), 33, 37, 39, 43, 47, 49, 53, 55, 57

## 10. Chapter 2.5: Continuity

(1) Quick Check: 1, 2, 4
(2) Exercise Set: 1-5, 7, 11-23(odd), 41, 43

## 11. Chapter 2.6: More Continuity

(1) Quick Check: 1-3
(2) Exercise Set: 17, 19, 21, 23, 46, 75

## 12. Chapter 3.1: Tangent Lines and Instantaneous Velocity

(1) Quick Check: 1, 3, 4, 5
(2) Exercise Set: 1, 2, 4, 7-16, 21

## 13. Chapter 3.2: The Derivative

(1) Quick Check: 1-4
(2) Exercise Set: 1-19(odd), 23, 35, 43, 44

## 14. Chapter 3.3: Techniques of Differentiation

(1) Quick Check: 1-5
(2) Exercise Set: 1-23(odd), 35, 37, 39

## 15. Chapter 3.4: The Product and Quotient Rule

(1) Quick Check: 1-5
(2) Exercise Set: 1-15(odd), 19-27(odd)

## 16. Chapter 3.5: The Derivatives of Trigonometric Functions

(1) Quick Check: 1-4
(2) Exercise Set: 1-25(odd), 29

## 17. Chapter 3.6: The Chain Rule

(1) Quick Check: 1-5
(2) Exercise Set: 7-25(odd), 45-53(odd)

## 18. Chapter 3.7: Related Rates

(1) Quick Check: 1-4
(2) Exercise Set: 1-17(odd)

## 19. Chapter 4.1: Implicit Differentiation

(1) Quick Check: 1-5
(2) Exercise Set: 1-31(odd)

## 20. Chapter 4.2: Derivatives of Logarithms

(1) Quick Check: 1, 2, 4
(2) Exercise Set: 1-25(odd)

## 21. Chapter 4.3: Derivatives of Exponential and Inverse Trig. Functions

(1) Quick Check: 1-5
(2) Exercise Set: 1-31(odd)

## Derivative Formulas

If $y = f(x)$, we will denote the derivative of $f$ in many ways. The following notations mean the same thing.

$$f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx}[f(x)]$$

To evaluate a derivative at a given value $x = c$, we will right one of the following equivalent notations.

$$f'(c) = y'(c) = \left.\frac{dy}{dx}\right|_{x=c}.$$

Let $f$ and $g$ be functions and $c$ be a constant. Let $y = f(x)$ and $u = g(x)$.

$\frac{d}{dx}[cf(x)] = cf'(x)$

$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$, Product Rule

$\frac{d}{dx}[f(x)/g(x)] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$, $g(x) \neq 0$, Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$, Chain Rule

$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$, Chain Rule

$\frac{d}{dx}[u^n] = nu^{n-1}\,\frac{du}{dx}$, Power Rule

$\frac{d}{dx}[\sin u] = \cos u\,\frac{du}{dx}$

$\frac{d}{dx}[\cos u] = -\sin u\,\frac{du}{dx}$

$\frac{d}{dx}[\tan u] = \sec^2 u\,\frac{du}{dx}$

$\frac{d}{dx}[\csc u] = -\csc u \cot u\,\frac{du}{dx}$

$\frac{d}{dx}[\sec u] = \sec u \tan u\,\frac{du}{dx}$

$\frac{d}{dx}[\cot u] = -\csc^2 u\,\frac{du}{dx}$

$\frac{d}{dx}[f^{-1}(u)] = \frac{1}{f'(f^{-1}(u))}\,\frac{du}{dx}$, $f$ is a one-to-one functions

$\frac{d}{dx}[e^u] = e^u\,\frac{du}{dx}$

$\frac{d}{dx}[a^u] = (\ln a)a^u\,\frac{du}{dx}$, $a > 1, a \neq 1$

$\frac{d}{dx}[\ln|u|] = \frac{1}{u}\,\frac{du}{dx}$

$\frac{d}{dx}[\log_a|u|] = \frac{1}{(\ln a)u}\,\frac{du}{dx}$, $a > 1, a \neq 1$

Notice that many of the above formulas are using the chain rule. In these formulas, if $u = x$, then the power rule states that $\frac{du}{dx} = \frac{dx}{dx} = 1$.

## 22. Chapter 5.1: Analysis of Functions I: Increase, Decrease, and Concavity

(1) Quick Check: 2-4
(2) Exercise Set: 1-27(odd), 57

## 23. Chapter 5.2: Analysis of Functions I: Relative Extrema; Graphing Polynomials

(1) Quick Check: 1-4
(2) Exercise Set: 1-41(odd)

## 24. Chapter 5.4: Absolute Maxima and Minima

(1) Quick Check: 1,2,4
(2) Exercise Set: 1-37(odd)

## 25. Chapter 5.5: Applied Maximum and Minimum Problems

(1) Quick Check: 1-4
(2) Exercise Set: 1, 3, 5, 9, 11, 15, 19, 25, 35, 39, 43

## 26. Chapter 5.7: Rolle's Theorem; Mean-Value Theorem

(1) Quick Check: 1,5
(2) Exercise Set: 1-11(odd), 19

## 27. Chapter 5.8: Rectilinear Motion

(1) Quick Check: 1-4
(2) Exercise Set: 1-17(odd), 29.

## 28. Applied Max/Min Examples

**Number 2(a):**

How should two nonnegative numbers be chosen so that their sum is 1 and the sum of their squares is as small as possible?

Step 1, identify the unknowns.

Let $x$ and $y$ be two nonnegative numbers.

Step 2, identify the quantity to be minimized.

$$\text{Let } S = x^2 + y^2.$$

Step 3, write an equation which relates $x$ and $y$.

$$x + y = 1.$$

Step 4, Write $S = S(x)$.

$$S = S(x) = x^2 + (1 - x)^2.$$

Step 5, Identify the domain of $S(x)$.

Since $x \geq 0$ and $y = 1 - x \geq 0$, $x$ must satisfy $0 \leq x \leq 1$.

Step 6, Find the absolute minimum of $S(x)$ over the interval $[0, 1]$.

Since $S(x)$ is a polynomial, all of its critical points are stationary points. So, solving

$$S'(x) = 0$$

gives us

$$2x - 2(1 - x) = 0$$

or

$$x = 1/2.$$

Since

$$S(0) = 1, \quad S(1/2) = 1/2, \quad \textit{and } S(1) = 1,$$

choosing $x = y = 1/2$ will produce a minumum value of $S = 1/2$.

**Number 6:**

A rectangle is to be insribed in a right triangle having sides of 6 in., 8 in., and 10 in. Finde the dimensions of the rectangle with the greatest area assuming the rectangle is positioned as in Figure Ex-6 on page 319.

Step 1, identify the unknown dimensions of the rectangle.

Let $x$ be the base of the rectangle and $y$ be the height.

Step 2, Identify the quantity to be maximized.

$$\text{Let } A = xy.$$

Step 3, Write an equation relating $x$ and $y$.

Using similar triangles we have

$$\tfrac{6-x}{y} = \tfrac{6}{8}.$$

Step 4, Use the equation from step 3 to write $A = A(x)$.

Since

$$\tfrac{6-x}{y} = \tfrac{6}{8},$$

we have

$$\tfrac{24-4x}{3} = y.$$

Therefore,

$$A = A(x) = x\tfrac{24-4x}{3}.$$

Step 5, Identify the domain of $A(x)$.

Since $x$ is the base of the rectangle in figure Ex-6, $x$ must satisfy

$$0 \leq x \leq 6.$$

Step 6, Find the absolute maximum of $A(x)$ over the interval $[0, 6]$.

Since $A(x)$ is a polynomial, all of the critical points of $A(x)$ are stationary points. So, solving

$$A'(x) = 0$$

gives us

$$8 - \frac{8x}{3} = 0$$

or

$$x = 3.$$

Since

$$A(0) = 0, \quad A(3) = 12, \quad and\ A(6) = 0,$$

the absolute maximum value of $A$ is 12 square inches when $x = 3$ and $y = 4$.

## 29. Chapter 6.2: The indefinite integral

(1) Quick Check: 2, 3
(2) Exercise Set: 1-29(odd), 39-43(odd)

## 30. Chapter 6.3: Integration by substitution

(1) Quick Check: 1-4
(2) Exercise Set: 1-49(odd)

## 31. Chapter 6.5: The definite integral

(1) Quick Check: 3
(2) Exercise Set: 11-21(odd)

## 32. Chapter 6.6: The Fundamental Theorem of Calculus

(1) Quick Check: 1,2
(2) Exercise Set: 1-31(odd), 61

## 33. Chapter 6.7: Rectilinear Motion

(1) Quick Check: 1-4
(2) Exercise Set: 1-17(odd), 31, 33, 37, 39

## 34. Chapter 6.8: Evaluating definite integrals by substitution

(1) Quick Check: 1-3
(2) Exercise Set: 1-47(odd), 55, 57, 62