Quiz 13 and 14 Solutions: Math 141 Spring 2008

6.2 #10:

(a)

$$\int x^{2/3}\ dx = 3x^{5/3}/5 + C$$

(b)

$$\int x^{-6}\ dx = -x^{-5}/5 + C$$

(c)

$$\int x^{-7/8}\ dx = 8x^{1/8} + C$$

6.5 #14:

(a) The region is a rectangle with base length 5 and height 6.

$$\int_{-10}^{-5} 6\ dx = (6)(5) = 30$$

(b) There are two parts of equal area, one above and one below the x-axis.

$$\int_{-\pi/3}^{\pi/3} \sin x\ dx = 0$$

(c) There are two triangles above the x-axis

$$\int_0^3 |x-2|\ dx = \tfrac{1}{2}(2)(2) + \tfrac{1}{2}(1)(1) = 2.5$$

(d) The region is one quarter of a circle with radius 2 and is above the x-axis.

$$\int_0^2 \sqrt{4-x^2}\ dx = \tfrac{1}{4}\pi(2)^2 = \pi$$

6.6 #16

If $F'(\theta) = \sec^2\theta$, then $F(\theta) = \tan\theta + C$. Therefore,

$$\int_0^{\pi/4} \sec^2\theta\ d\theta = \tan(\pi/4) - \tan 0 = 1$$

6.6 #24

If $F'(x) = \frac{1}{x\sqrt{x^2-1}}$, then $F(x) = -\sec^{-1} x + C$. Therefore,

$$\int_{-\sqrt{2}}^{-2/\sqrt{3}} \frac{1}{x\sqrt{x^2-1}}\, dx = -\sec^{-1}(-2/\sqrt{3}) + \sec^{-1}(\sqrt{2}) = -5\pi/6 + 3\pi/4 = -\pi/12$$

6.3 #12

Let $u = x/3$. Then $\frac{du}{dx} = 1/3$.

$$\int \cos(x/3)\ dx = 3\int \cos u\ du = 3\sin u + C = 3\sin(x/3) + C$$

6.3 #26

Let $u = x^4$. Then $\frac{du}{dx} = 4x^3$.

$$\int x^3 e^{x^4}\ dx = \frac{1}{4}\int e^u\ du = \frac{1}{4}e^u + C = \frac{1}{4}e^{x^4} + C$$

6.8 #14

Let $u = x/4 - 1/4$. Then $\frac{du}{dx} = 1/4$.

$$\int_{1-\pi}^{1+\pi} \sec^2(x/4 - 1/4)\, dx = 4\int_{-\pi/4}^{\pi/4} \sec^2 u\, du = 4\tan(\pi/4) - 4\tan(-\pi/4) = 8$$