Problem Set 3

MATH 777, Spring 2008, Cooper

Expiration: Friday, April 4

Each problem is worth 20 points. Rigorous proofs are required for all claims, although elegance and concision are nearly as important. You may only use results we proved in class, or which can reasonably be considered prerequisite material for this class, unless otherwise stated. You will receive 5 bonus points for submitting your solutions in LaTeX. **NOTE:** The text has been updated on the website, so the problem numbers below refer to the *third* edition.

1. Diestel §7, #3.
2. Diestel §7, #9.
3. Diestel §7, #10 & #11.
4. Diestel §7, #12.
5. Show that, for every graph H and $\epsilon > 0$, there is a $c > 0$ so that every graph G on n vertices, with n sufficiently large and $H \not\subseteq G$, contains a vertex set $X \subset V(G)$ so that $|X| \geq cn$ and $\|G[X]\| \leq \epsilon |X|^2$. In other words, every graph that does not contain H has a linear-sized "nearly empty" induced subgraph. (Hint: Apply Ramsey's Theorem to the regularity graph of G. Be careful about the quantifiers!)
6. Show that "exceptional pairs" are unavoidable in the statement of the Regularity Lemma by considering the "half graph" Γ defined by $V(\Gamma) = [2] \times [n]$, and $(1, i) \sim (2, j)$ iff $i < j$.