Math 544, Exam 1 Information

Exam 1 will be based on:

- Sections 1.1 - 1.3, 1.5 - 1.7, and 1.9.
- The corresponding assigned homework problems (see http://www.math.sc.edu/∼boylan/SCCourses/math5443/544.html). **At minimum, you need to understand how to do the homework problems.**
- Lecture notes: 8/24 - 9/21.

Topic List (not necessarily comprehensive):

You will need to know how to define vocabulary words/phrases defined in class.

§**1.1**: Matrix representation of a linear system: coefficient matrix, augmented matrix, elementary row operations, row equivalence.

§**1.2**: Solving linear systems via **Gauss-Jordan elimination**: echelon and reduced echelon forms of a matrix, identifying dependent and independent variables, recognizing when a system is consistent/inconsistent.

§**1.3**: Relationship between the number of nonzero rows, the number of leading 1's, and the number of columns in an augmented matrix in **reduced echelon form**. Homogeneous linear systems. The number of possible solutions to (for example)

1. a general linear system.
2. an $m \times n$ system with $m < n$.
3. a homogeneous system.

§**1.5**: Matrix operations: addition, multiplication, multiplication by scalars, dot product in $\mathbb{R}^n$.

§**1.6**: Properties of matrix addition, multiplication, and multiplication by scalars. The matrix transpose and its properties. What is the transpose of a product? What is a symmetric matrix? Also vector norm (length) in terms of the dot product.

§**1.7**: Linear combinations, linear dependence/independence: determinination of whether a given set of vectors is linearly dependent/independent. Non-singular matrices (remember, only square matrices can be singular or non-singular!); conditions equivalent to non-singularity of $A \in \text{Mat}_{n\times n}(\mathbb{R})$:

1. $Ax = \theta$ has only the trivial solution $x = \theta$
2. columns of A are linearly independent
3. $\forall b \in \mathbb{R}^n$, $Ax = b$ has a unique solution.

4. A is invertible.
5. A is row equivalent to the identity, I_n.

§**1.9**: Matrix inverses: existence of inverses (see above, e.g., A is invertible $\iff$ A is non-singular), using inverses to solve systems, computing inverses by row reduction, formula for inverse of 2×2 matrix, algebraic properties of inverses (e.g., what is the inverse of a product of two matrices?) Remember, only square matrices can be invertible.

Notes:

- Only **sets of vectors** can be **linearly dependent/independent**. It does not make sense to speak of matrices or systems of equations being linearly dependent/independent.
- Only **systems of linear equations** can be **consistent/inconsistent**. It does not make sense to speak of matrices or sets of vectors as consistent/inconsistent.
- Only **squares matrices** can be **singular/non-singular**. It does not make sense to speak of a system of equations or set of vectors as being singular/non-singular. The same applies for **invertibility**.