

Math 544, Exam 2 Information

Exam 2 will be based on:

- Sections 3.2 - 3.7.
- The corresponding assigned homework problems
(see <http://www.math.sc.edu/~boylan/SCCourses/math5442/544.html>).
At minimum, you need to understand how to do the homework problems.
- Lecture notes: 2/16 - 3/9.

Topic List (not necessarily comprehensive):

You will need to know how to define vocabulary words/phrases defined in class.

§3.2: Vector space axioms:

- closure properties (2)
- addition properties (4)
- multiplication properties (4)

Subspaces: How do you determine whether a subset W of a vector space V is a subspace?

1. To show that W is a subspace, you need to verify the two subspace axioms.
2. To show that W is not a subspace, it suffices to provide an example in which one of the axioms is violated.

§3.3: Examples of subspaces: What is the **Span** of a set of vectors?

Let $A \in \text{Mat}_{m \times n}$. Important subspaces associated to A are:

- The **null space** of A (a subspace of \mathbb{R}^n).
- The **range** of A (a subspace of \mathbb{R}^m).
- The **column space** of A (a subspace of \mathbb{R}^m which coincides with the range of A).
- The **row space** of A (a subspace of \mathbb{R}^n).

Problem: Given a matrix A , provide an algebraic description of the vectors in each of the above subspaces.

Fact: If A and B are row equivalent $m \times n$ matrices, then they have the same row space.

§3.4: Bases for subspaces: What is a **basis** for a subspace W of \mathbb{R}^n ?

Fact: If B is a basis for W , then every vector $w \in W$ has a unique representation as a linear combination of vectors from B .

Given a matrix $A \in \text{Mat}_{m \times n}$, compute bases for the following subspaces:

- **Row(A):** To compute a basis for $\text{Row}(A)$, put A in echelon form, B . The nonzero rows of B form a basis for $\text{Row}(A)$.
- **Col(A):** There are two methods.
 1. If you want a basis **consisting of columns of A** , you put A in reduced echelon form, B . The columns of B with the leading 1's correspond to the columns of A which comprise the basis.
 2. If you do not require your basis to be a subset of the column vectors of A , you can compute a basis for $\text{Col}(A)$ by computing a basis for $\text{Row}(A^T) = \text{Col}(A)$. To do this, put A^T in echelon form. The non-zero rows of the echelon form give the desired basis vectors.
- **Range(A):** Since the range and column space of A agree, you compute $\text{Range}(A)$ just as you would $\text{Col}(A)$.
- **Null(A):** $\text{Null}(A)$ is the set of solutions to the homogenous system $A\vec{x} = \vec{0}$. Therefore, begin by solving $A\vec{x} = \vec{0}$. Convert your solution to vector form. i.e., write your solution as $\text{Null}(A) = \text{Span}(B)$. Verify that the vectors in B are linearly independent.

§3.5: **Dimension:** Let W be a subspace of \mathbb{R}^n .

Facts:

- W has many bases, but the number of vectors in every basis is the same. This number is called the **dimension** of W .
- If $A \in \text{Mat}_{m \times n}$, then $\text{rank}(A) + \text{nullity}(A) = n$.
(what is $\text{rank}(A)$? $\text{nullity}(A)$?).
- $\text{rank}(A) = \dim(\text{Col}(A)) = \dim(\text{Row}(A))$.
- $A \in \text{Mat}_{n \times n}$ is non-singular if and only if $\text{rank}(A) = n$ and $\text{nullity}(A) = 0$.

Know Theorem 3.9, page 207.

§3.6: **Orthogonal bases for subspaces:** Define the terms **orthogonal** and **orthonormal**. How do you convert an orthogonal set to an orthonormal one? If B is a basis for a subspace W of \mathbb{R}^n consisting of orthogonal vectors, how do you express a vector $\vec{w} \in W$ as a linear combination of basis vectors? Must a basis of orthogonal vectors for W **always** exist?

§3.7: **Linear transformations from \mathbb{R}^n to \mathbb{R}^m :** Define what it means for a function $T : \mathbb{R}^n \mapsto \mathbb{R}^m$ to be a linear transformation. Given a function $T : \mathbb{R}^n \mapsto \mathbb{R}^m$, how does one check whether T is a linear transformation?

Fact: If $T : \mathbb{R}^n \mapsto \mathbb{R}^m$ is a linear transformation, then for every $\vec{x} \in \mathbb{R}^n$, $T(\vec{x}) = A\vec{x}$, where

$$A = [T(\vec{e}_1) \mid \cdots \mid T(\vec{e}_n)]$$

(i.e., it suffices to know what T does to the standard basis for \mathbb{R}^n).

What are the **Null space** and **Range** of T ?