Math 547/702I, Homework 4 Problems.

You are expected to do every assigned homework problem. You should review the relevant sections of the text and review the relevant class notes. Your solutions must be clearly written and neatly prepared. In particular, your pages must be stapled together, and must not have rough edges.

Due 2/22: Beachy and Blair §9.1: 3, 10, 13.

§9.2: 6.

Extra problems:. Attempt these only after you have tried the problems above.
You can try some, all, or none of the extra problems.

§9.1: 11, 14.

Notes on these problems:

1. **#11 a**: Here are some steps to a solution.
   (a) Show that $\gcd(a, b) = 1$ implies that $\gcd(a, n) = 1 = \gcd(b, n)$. In particular, a and b are invertible modulo n.
   (b) What must be true of **any** ring homomorphism $\phi : \mathbb{Z}[i] \to \mathbb{Z}_n$ with $a + bi \in \ker\phi$? In particular, what must $\phi(1)$ be? What must $\phi(i)$ be?
   (c) To prove that the ϕ constructed using the logic in (2) has kernel $(a + bi)$, try to write down a **single condition modulo** n on integers x, y which guarantees that $x + iy \in (a + bi)$.
2. **#14**: First prove that for any positive M, there are finitely many $a + b\sqrt{2} \in S$ such that **both** $|a + b\sqrt{2}|$ and $|a - b\sqrt{2}| < M$. Now use this fact together with #13 to do the problem.