Math 374. Exam 3. 4/17/09. **Name:** ______________

- Read problems carefully. Show all work.
- No notes, calculator, or text.
- The exam is approximately 15 percent of the total grade.
- There are 100 points total. Partial credit may be given.
- Do not multiply out high powers, factorials, or binomial symbols.

1. **(20 points) Short answer.**

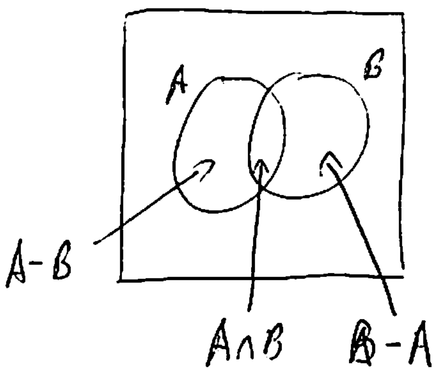
   (a) **(10 points)** Let $\emptyset$ denote the empty set, and let $S = \{\emptyset,\ \{\emptyset\}\}$. Find the **power set** of S.

$$\{\phi, \{\phi\}, \{\{\phi\}\}, \underbrace{\{\phi, \{\phi\}\}}_{=S}\}$$

   (b) **(10 points)** Let A and B be sets. Suppose that

$$A - B = \{1, 5, 7, 8\},$$
$$A \cap B = \{3, 6, 9\},$$
$$B - A = \{2, 4, 10\}.$$

   Find the sets A and B. It may help to draw a **Venn Diagram.**



$A = \{1, 3, 5, 6, 7, 8, 9\}.$

$B = \{2, 3, 4, 6, 9, 10\}.$

2. **(20 points) Short answer.**

(a) **(10 points)** A jar has 5 red marbles and 7 blue marbles. Find the number of ways to select 3 marbles of the same color from the jar. (In other words, we want the number of ways to select either 3 red marbles **or** 3 blue marbles.)

$$\binom{5}{3} + \binom{7}{3}$$

(b) **(10 points)** A committee of size 7 is to be formed from people in groups A and B. Group A has size 11, and group B has size 14. How many ways are there to form the committee if **at least one member** must come from group A?

$$\binom{25}{7} - \binom{14}{7}$$

total # ways

ways with no one from group A.

3. **(10 points)** Each house in a row of n houses is assigned a distinct number between 25 and 100 inclusive. What is the smallest value of n needed to ensure that there are at least two houses numbered consecutively? Use the **Pigeonhole Principle.**

$100 - 25 + 1 = 76$, $\frac{76}{2} = 38$; We need $\boxed{39}$

$\underbrace{\{25, 26\}, \ldots \quad \{99, 100\}.}_{38 \text{ boxes.}}$

4. **(10 points)** In a survey of 100 college students, it is found that

- 33 like brussels sprouts, $\leftarrow S$
- 29 like broccoli, $\leftarrow B$
- 41 like cauliflower, $\leftarrow C$
- 15 like brussels sprouts and broccoli, $S \cap B$
- 18 like brussels sprouts and cauliflower, $S \cap C$
- 23 like broccoli and cauliflower, and $B \cap C$
- 9 like all three vegetables. $\leftarrow S \cap B \cap C$.

How many of the 100 students **do not like any** of these vegetables?

Use the **Principle of Inclusion and Exclusion.**

$$|S \cup B \cup C| = \underbrace{33+29+41}_{103} - (15+18+23) + 9$$

$$103 - 56 + 9 = \boxed{56}$$

$$100 - 56 = \boxed{44}$$

5. **(20 points) Short answer.**

(a) **(10 points)** How many ways can 17 doctors and 12 lawyers stand in line so that no two lawyers stand next to each other?

$$17! \cdot P(18,12) = 17! \cdot \frac{18!}{6!}$$

(b) **(10 points)** How many distinct permutations of the letters in the word "RECURRENCE" start with the letter "N"?

$\underline{N}\ \underbrace{\_\ \_\ \_\ \_}_{9\text{ others.}}$ $$\frac{9!}{3!\,3!\,2!\,1!}$$

6. **(20 points) Short answer.**

(a) **(10 points)** Find the coefficient of x^6y^3 in $(2x-5y)^9$. Use the **Binomial Theorem.**

$$n = 9$$

$$\underbrace{\binom{9}{k}(2x)^k(-5y)^{9-k}}_{k\text{th term}}, \quad k = 6: \quad \binom{9}{6} 2^6 (-5)^3$$

(b) **(10 points)** Use the **Binomial Theorem** to show that

$$\sum_{k=0}^{n}\binom{n}{k}\cdot\frac{1}{2^k} = \binom{n}{0}+\binom{n}{1}\cdot\frac{1}{2}+\cdots+\binom{n}{k}\cdot\frac{1}{2^k}+\cdots+\binom{n}{n}\cdot\frac{1}{2^n} = (3/2)^n.$$

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}.$$ Let $x = \frac{1}{2}$, $y = 1$.