HW Sols., §4.4.

2. a) Is a function, 1-1, not onto

   b) not a function

   c) 1-1, onto

   d) onto, not 1-1

5. a) $f(\mathbb{N}) = \{0, 1, 4, 9, 16, \ldots\}$

   b) $f(\mathbb{Z}) = f(\mathbb{N})$

   c) $f(\mathbb{R}) = \{x \mid x \in \mathbb{R},\ x \geq 0\}$ $(\mathbb{R}^+ \cup \{0\})$

7 a) not a function from S to T

   b) is a function (not 1-1, or onto)

   c) 1-1, and onto

   d) not a function from S to T

   e) not a function

11. $n = 2k+1$ (odd)

18 a) $f = \{(a,x), (b,x), (c,y), (d,y)\}$

   b) $f = \{(a,x), (b,x), (c,y), (d,z)\}$

   c) no

19 x an integer

21 Let $n = \lceil x \rceil$. Then n is an integer such that $n-1 < x \le n$. Now we add 1, through the inequality so that $n < x+1 \le n+1$ so that $n+1$ is the largest integer so that $n+1 = \lceil x+1 \rceil$.

29 $g \circ f = \{(1,6), (2,7), (3,9), (4,9)\}$

32 a) $g \circ f = 12x^3$, $f \circ g = 48x^3$

b) $g \circ f = x^2 - 2x + 1$, $f \circ g = (4x^2 - 1)/2$

c) $g \circ f = \lceil x \rceil$, $f \circ g = \lfloor x \rfloor$

33 a) If $f(s_1) = f(s_2)$ then $g(f(s_1)) = g(f(s_2))$ so that $g \circ f(s_1) = g \circ f(s_2)$. Then since $g \circ f$ is 1-1, and $s_1 = s_2$ and therefore f is 1-1.

b) For $u \in U$, $\exists s \in S$ so that $(g \circ f)(s) = u$, since $g \circ f$ is onto. Thus $g(f(s)) = u$ and $f(t) \in T$, that is, the preimage of u under g, so g is onto.

c), d) Let $S = \{1,2,3\}$, $T = \{1,2,3,4\}$ $U = \{1,2,3\}$

Then $f = \{(1,1), (2,2), (3,3)\}$, $g = \{(1,1), (2,2), (3,3), (4,3)$

let $f: S \to T$, $g: T \to U$

36 $f^{-1}: T \to S$, $g^{-1}: U \to T$, so that $f^{-1} \circ g^{-1} : U \to S$

For $s \in S$, let $f(s) = t$ and $g(t) = u$. Then

$(f^{-1} \circ g^{-1})(g \circ f)(s) = s$. Also, for $u \in U$ $(g \circ f)(f^{-1} \circ g^{-1})(u) = u$

Then, $(f^{-1} \circ g^{-1})(g \circ f) = i_S$ and $(g \circ f)(f^{-1} \circ g^{-1}) = i_U$

so that $f^{-1} \circ g^{-1} = (g \circ f)^{-1}$

66 $$\lim_{x \to \infty} \frac{3x^3 - 7x}{\frac{1}{2}x^3} = \lim_{x \to \infty} \frac{9x^2 - 7}{\frac{3}{2}x^2} = \lim_{x \to \infty} \frac{18x}{3x} = 6$$

68 $$\lim_{x \to \infty} \frac{\log x}{x} = \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = 0$$

71 $[\log x] = [\ln x] < [2(\log x)] < [\sqrt{x}] < [x] < [x \log x]$

$< [x^3] = [2x^3 + x] = [x^3 + \log x] < [e^x]$