Math 241. Quiz 9. 4/8/09. **Name:** ____________________

- Read problems carefully. Show all work.
- There are 15 points total.

1. **§14.9, #9 (10 points):** Use Lagrange multipliers to find the maximum and minimum values (absolute) of

$$f(x, y, z) = 2x + y - 2z$$

subject to the constraint

$$g(x, y, z) = x^2 + y^2 + z^2 - 4 = 0.$$

$$\nabla f = 2\vec{i} + \vec{j} - 2\vec{k} \; ; \; \nabla g = (2x)\vec{i} + (2y)\vec{j} + (2z)\vec{k}$$

$$\nabla f = \lambda \nabla g \iff \left\{\begin{array}{l} 2 = 2\lambda x \iff \frac{1}{x} = \lambda \\ 1 = 2\lambda y \iff \frac{1}{2y} = \lambda \\ -2 = 2\lambda z \iff -\frac{1}{z} = \lambda \end{array}\right\} \Longrightarrow$$

$$\left.\begin{array}{l} \frac{1}{x} = \frac{1}{2y} \iff x = 2y \\ \frac{1}{2y} = -\frac{1}{z} \iff z = -2y \end{array}\right\} \xrightarrow[\text{in } g]{\text{subst.}} g(x,y,z) = (2y)^2 + y^2 + (-2y)^2 - 4 = 0$$

$$\iff 9y^2 = 4 \iff y^2 = \frac{4}{9} \iff y = \pm\frac{2}{3}$$

$$\left.\begin{array}{l} y = 2/3 \to x = 4/3, \; z = -4/3 \\ y = -2/3 \to x = -4/3, \; z = 4/3 \end{array}\right\} \Rightarrow \text{pts are: } P_1(4/3, 2/3, -4/3), \; P_2(-4/3, -2/3, 4/3)$$

	P_1	P_2
$f(x,y,z)$	6 ← max	-6 ← min.

2. **§15.1, #5 (5 points):** Evaluate the iterated integral

$$\int_0^{\ln 3} \int_0^{\ln 2} e^{x+y}\, dy\, dx.$$

$$u = x+y \Rightarrow I = \int_0^{\ln 3} \left(\int_x^{x+\ln 2} e^u \, du \right) dx$$

$$= \int_0^{\ln 3} \left(e^u \Big|_{u=x}^{x+\ln 2} \right) dx = \int_0^{\ln 3} \Big(\underbrace{e^{x+\ln 2} - e^x}_{= e^x(e^{\ln 2}-1) = e^x(2-1) = e^x} \Big) dx$$

$$= \int_0^{\ln 3} e^x \, dx = e^x \Big|_{x=0}^{\ln 3} = e^{\ln 3} - e^0 = 3 - 1 = \boxed{2}.$$