Math 241, Exam 2 Information

Exam 2 will be based on:

- Sections 13.3, 13.4, 13.6, 14.1 - 14.7
- The corresponding assigned homework problems
(see http://www.math.sc.edu/∼boylan/SCCourses/241Sp09/241.html)
At minimum, you need to understand how to do the homework problems.
- Lecture notes: 2/9 - 3/6.

Topic List (not necessarily comprehensive):

You will need to know how to define vocabulary words/phrases defined in class.

§**13.3**: **Change of parameter; arc length**. How do you compute arc length on a curve defined by parametric equations? If you are given parametric equations for a curve: $x(t)$, $y(t)$, $z(t)$, how do you change to a parametrization using arc length, s, as the parameter?

§**13.4**: **Unit tangent, normal, and binormal vectors**. Give the definitions of unit tangent and unit normal, and be able to compute these. They are orthogonal!

§**13.6**: **Motion along a curve**. The motion of a particle is given by a vector-valued function $\vec{\mathbf{r}}(t)$. How do you compute its velocity? acceleration? speed? What is the relationship between velocity, speed, and the unit tangent vector? If you are given acceleration, $\vec{\mathbf{a}}(t)$ and the velocity at time zero, $\vec{\mathbf{v}}(0)$, how do you find velocity $\vec{\mathbf{v}}(t)$? Similarly, how can you get $\vec{\mathbf{r}}(t)$ from $\vec{\mathbf{v}}(t)$?

§**14.1**: **Functions of two or more variables**. Given $z = f(x, y)$, be able to sketch (or describe in words) the domain.

§**14.2**: **Limits and continuity**. Be able to compute limits of functions of 2 or 3 variables **along curves** C defined by parametric equations. **General limits**: If a function f is continuous at P, its limit as you approach P is simply $f(P)$. To show that a general limit does not exist, you could either

- Find a curve C through P for which the limit along C does not exist, or
- Find two curves C_1 and C_2 such that the limits along these curves disagree.

§**14.3**: **Partial derivatives**. Be able to compute partial derivatives of functions of 2, 3, or possibly more variables. Given, for example, $z = f(x, y)$, what is the geometric meaning of $f_x(x_0, y_0)$? $f_y(x_0, y_0)$? Be able to use both the "subscript" notation and the "∂" notation. Equality of mixed partials: If $z = f(x, y)$ has f_x and f_y continuous near (x_0, y_0), then $f_{xy}(x_0, y_0) = f_{yx}(x_0, y_0)$.

§**14.5**: **The chain rule**. Be able to do ordinary and partial derivative computations using the chain rule. **Important:** the particular form that the chain rule takes depends on how the variables in question depend on each other. As such, it is can be very useful to **draw a tree diagram** to show the variable dependencies.

§**14.6**: **Directional derivatives and gradients**. What does the directional derivative of $z = f(x, y)$ in the direction of $\vec{\mathbf{u}}$ at the point $P(x_0, y_0)$ mean geometrically? This is written $D_{\vec{\mathbf{u}}}f(x_0, y_0)$. It is the rate of change of f with respect to distance from P in the direction of $\vec{\mathbf{u}}$ (so it is a **scalar**). How does one compute $D_{\vec{\mathbf{u}}}f(x_0, y_0)$? The formula can be written as

$$D_{\vec{\mathbf{u}}}f(x_0, y_0) = f_x(x_0, y_0)u_1 + f_y(x_0, y_0)u_2 = \nabla f(x_0, y_0) \cdot \vec{\mathbf{u}},$$

where $\nabla f(x_0, y_0)$ is the **gradient vector** of f at (x_0, y_0). The gradient points in the direction of "steepest" ascent on the surface f (i.e., in the direction for which the directional derivative is a maximum). Similarly, the direction opposite the gradient is the direction of "steepest descent" on the surface f (i.e., the direction for which the directional derivative is a minimum).

§**14.7**: **Tangent planes and normal vectors**. Given a surface S and a point $P(x_0, y_0, z_0)$ on the surface, be able to find the equation of the plane tangent to the surface through P and the equation of the line normal to the surface through P. If the surface has equation $z = f(x, y)$, then the tangent plane through P is

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

and the normal line is

$$x = x_0 + f_x(x_0, y_0)t, \;\; y = y_0 + f_y(x_0, y_0)t, \;\; z = f(x_0, y_0) - t.$$

If the surface has equation $F(x, y, z) = c$ (defined implicitly), then the the tangent plane through P is

$$F_x(P)(x - x_0) + F_y(P)(y - y_0) + F_z(P)(z - z_0) = 0;$$

the gradient vector $\nabla F(P)$ is normal to this plane.