Quiz 2
January 17, 2008

Name________________________________

Instructions. Complete the following exercises to the best of your ability. Please show all work in an organized and legible manner Do not use a calculator. You may use the back of this page if you need more room; however please indicate when you have done so.

1. Evaluate the definite integral

$$\int_{\ln 2}^{\ln\left(\frac{2}{\sqrt{3}}\right)} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}dx.$$

Substitute $u = e^{-x}$ so that, from the chain rule, $du = -e^{-x}dx$ whence $dx = -\frac{1}{u}du$. Integrating with respect to u, the upper and lower bounds become

$$\begin{aligned} e^{-\ln 2} &= \frac{1}{2} \quad \text{and} \\ e^{-\ln\left(\frac{2}{\sqrt{3}}\right)} &= \frac{\sqrt{3}}{2} \end{aligned}$$

respectively. Therefore

$$\begin{aligned} \int_{\ln 2}^{\ln\left(\frac{2}{\sqrt{3}}\right)} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}dx &= -\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-u^2}}du \\ &= -\left(\sin^{-1} u\right)\Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \\ &= -\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)\right) \\ &= -\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = -\frac{\pi}{6}. \end{aligned}$$