

Quiz 1
January 14, 2008

Name _____

Instructions. Complete the following exercises to the best of your ability. Please show all work in an organized and legible manner. Do not use a calculator. You may use the back of this page if you need more room; however please indicate when you have done so.

1. Evaluate the indefinite integral

$$\int 3x^2(1+x^3) dx.$$

Method 1:

$$\begin{aligned}\int 3x^2(1+x^3) dx &= \int 3x^2 + 3x^5 dx = 3 \int x^2 dx + 3 \int x^5 dx \\ &= 3 \left(\frac{1}{3} x^3 \right) + 3 \left(\frac{1}{6} x^6 \right) + C \\ &= x^3 + \frac{1}{2} x^6 + C\end{aligned}$$

Method 2: Substitute $u = 1 + x^3$ whereby $dx = \frac{1}{3x^2} du$ and hence

$$\begin{aligned}\int 3x^2(1+x^3) dx &= \int u du = \frac{1}{2} u^2 + C_1 \\ &= \frac{1}{2} (1+x^3)^2 + C_1 \\ &= x^3 + \frac{1}{2} x^6 + \left(\frac{1}{2} + C_1 \right) \\ &= x^3 + \frac{1}{2} x^6 + C.\end{aligned}$$

2. Evaluate the definite integral

$$\int_0^1 (5x^4 + 2x^3 + 3x^2 + x + 1) dx.$$

$$\begin{aligned}\int_0^1 (5x^4 + 2x^3 + 3x^2 + x + 1) dx &= \left(5 \int x^4 dx + 2 \int x^3 dx + 3 \int x^2 dx + \int x dx + \int 1 dx \right) \Big|_0^1 \\ &= \left(5 \cdot \frac{1}{5} x^5 + 2 \cdot \frac{1}{4} x^4 + 3 \cdot \frac{1}{3} x^3 + \frac{1}{2} x^2 + x \right) \Big|_0^1 \\ &= 1 + \frac{1}{2} + 1 + \frac{1}{2} + 1 - 0 = 4.\end{aligned}$$