

Problem 1. Use the definition of **definite integral** to express the integrals as limits.

- (a) $\int_1^2 2x \, dx$
 (b) $\int_0^1 \frac{x}{x+1} \, dx$
 (c) $\int_1^2 \sqrt{x} \, dx$
 (d) $\int_{-\pi/2}^{\pi/2} (1 + \cos x) \, dx$

Problem 2. Sketch the region whose signed area is represented by the following definite integral, and evaluate the integral using an appropriate formula from geometry.

- (a) $\int_0^3 x \, dx$
 (b) $\int_0^2 (1 - \frac{1}{2}x) \, dx$
 (c) $\int_0^5 2 \, dx$
 (d) $\int_{-1}^2 |2x - 3| \, dx$

Problem 3. Use the properties of definite integrals and appropriate formulas from geometry to evaluate the following integrals.

- (a) $\int_{-1}^3 (4 - 5x) \, dx$
 (b) $\int_{-2}^2 (1 - 3|x|) \, dx$
 (c) $\int_0^1 (x + 2\sqrt{1-x^2}) \, dx$
 (d) $\int_{-3}^0 (2 + \sqrt{9-x^2}) \, dx$

Problem 4. Find $\int_{-1}^2 (f(x) + 2g(x)) \, dx$ if $\int_{-1}^2 f(x) \, dx = 5$ and $\int_{-1}^2 g(x) \, dx = -3$.

Problem 5. Find $\int_1^4 (3f(x) - g(x)) \, dx$ if $\int_1^4 f(x) \, dx = 2$ and $\int_1^4 g(x) \, dx = 10$.

Problem 6. Evaluate the following limits by expressing them as a definite integral over the interval $[a, b]$, and applying appropriate formulas from geometry.

- (a) $\lim_{n \rightarrow \infty} \sum_{k=1}^n (3x_k^* + 1) \Delta x; a = 0, b = 1.$
 (b) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{4 - (x_k^*)^2} \Delta x; a = -2, b = 2.$