Edge detection using the Hidden Markov Tree Model for the Complex Wavelet Transform

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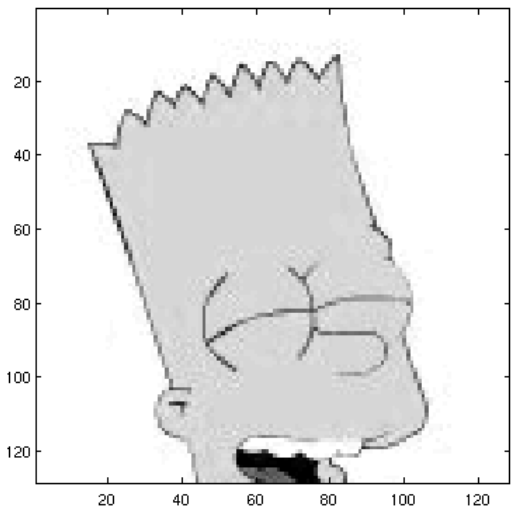
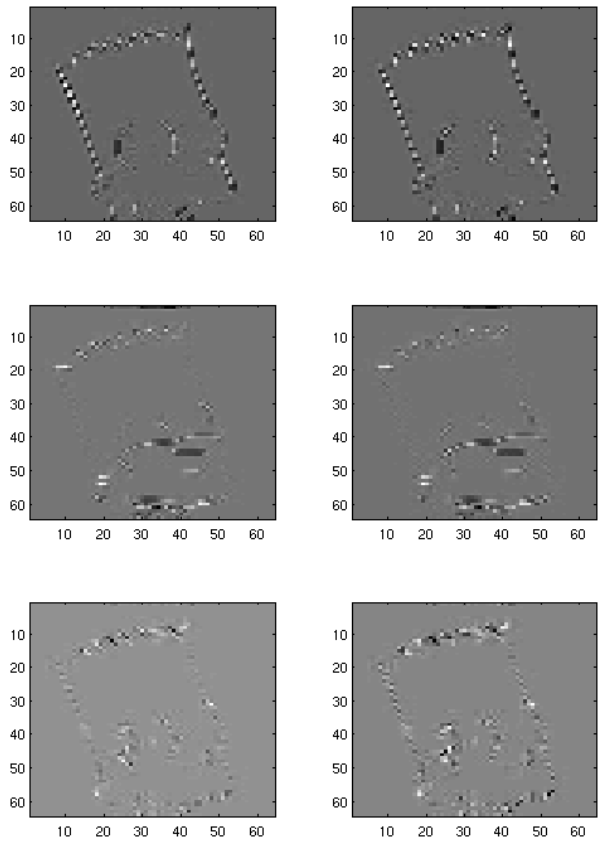
Dual-Tree Complex Wavelet Transform

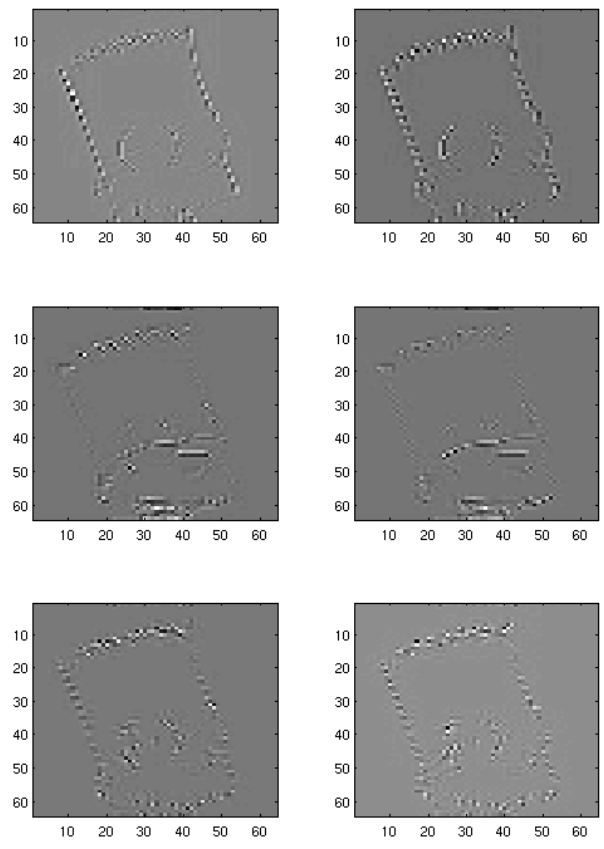


Original Image

Coefficients at finest scale (6 directions)



Real part



Imaginary part

The Wavelet-domain hidden Markov Tree

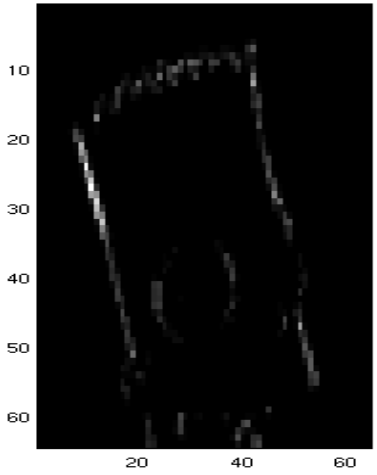
To each wavelet coefficient $w_k = u_k + iv_k$, associate a discrete hidden state q_k that takes on values $m = \mathtt{S}, \mathtt{L}$ with probability mass function $p(q_k)$.
Conditioned on $q_k = m$, w_k is Gaussian with mean $\mu_{k,m}$ and variance $\sigma^2_{k,m}$. Thus, overall marginal `pdf` is

$$g(w_k) = \sum_{m=\mathtt{S},\mathtt{L}} p(q_k = m) g(w_k | q_k = m),$$

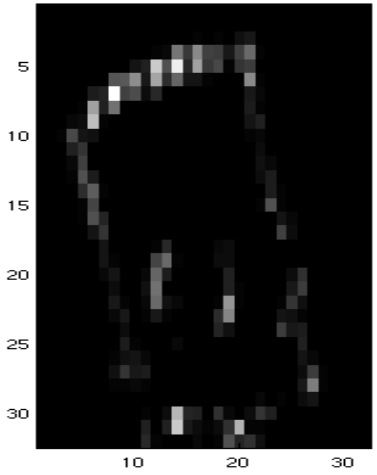
with $g(w_k | q_k = m) \approx \mathcal{N}(\mu_{k,m}, \sigma_{k,m})$.

Edge detection

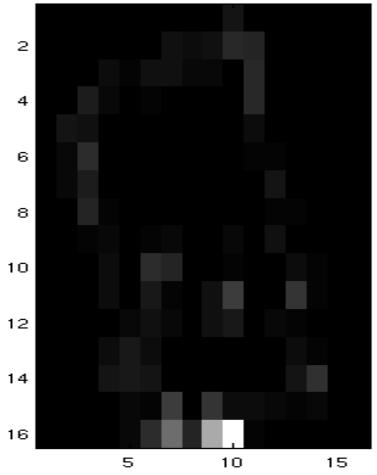
magnitude $|w_k|$



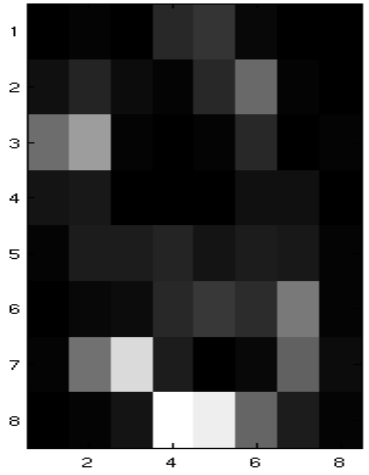
scale 1



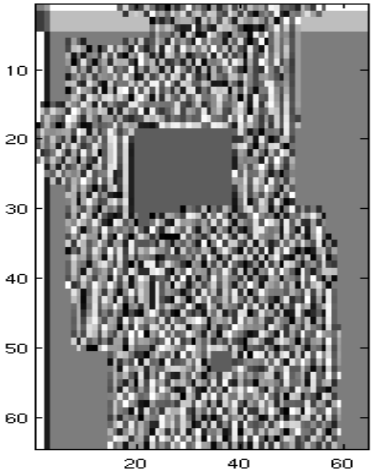
scale 2

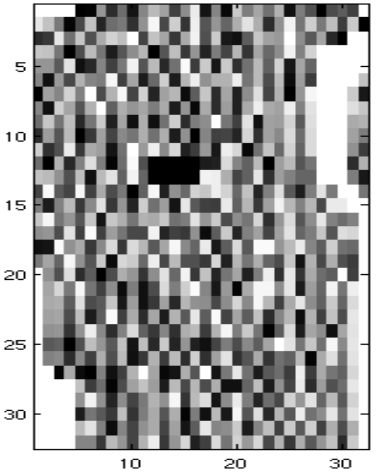


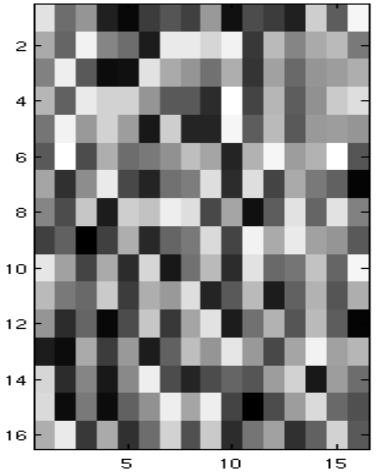
scale 3

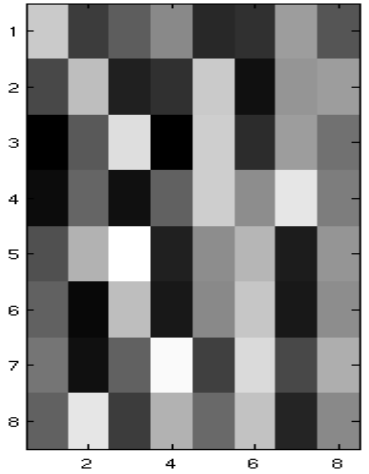


scale 4





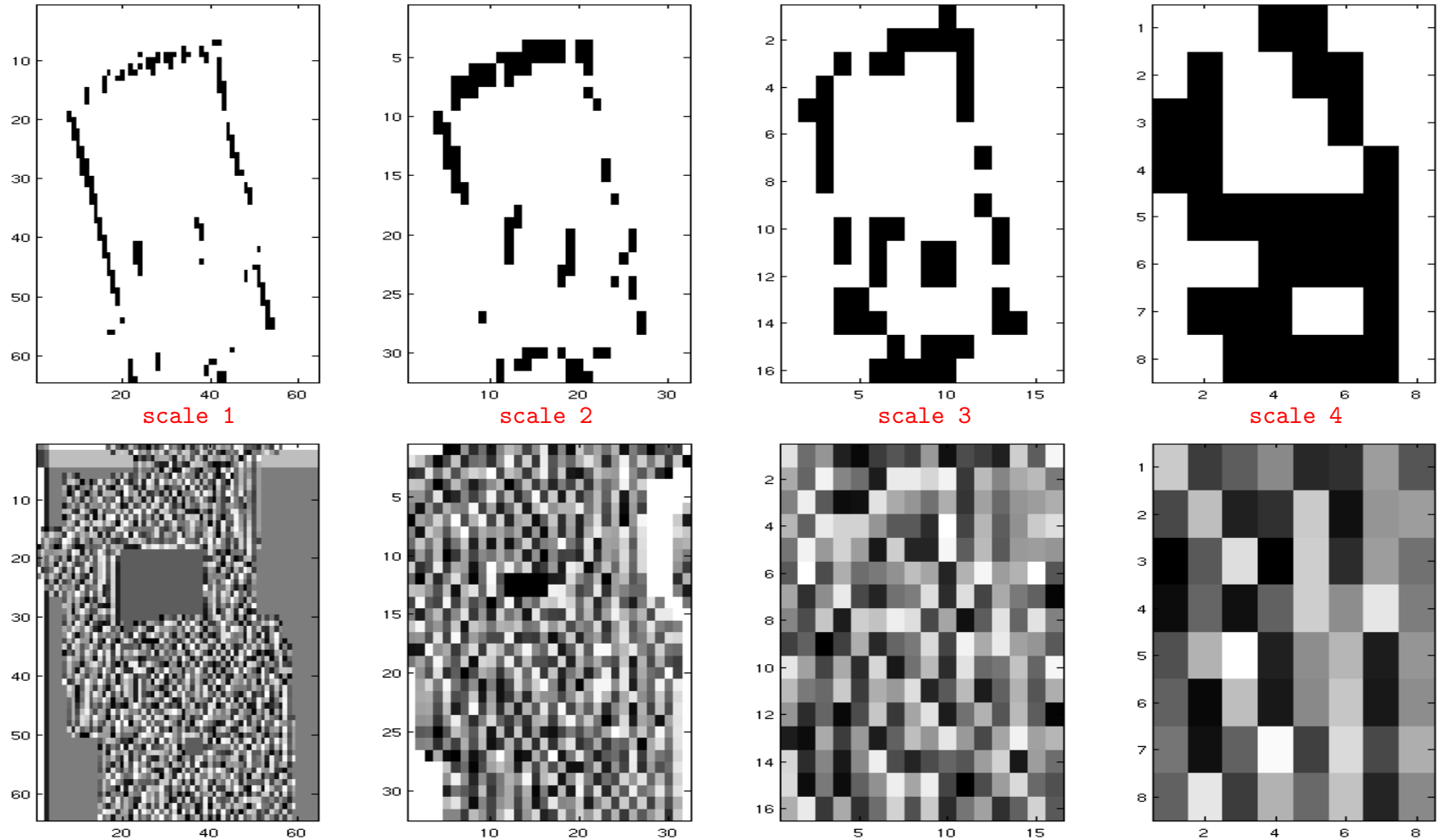




angle $\angle w_k$

Edge detection

S/L-state (S = white, L = black)



angle $\angle w_k$

Edge detection

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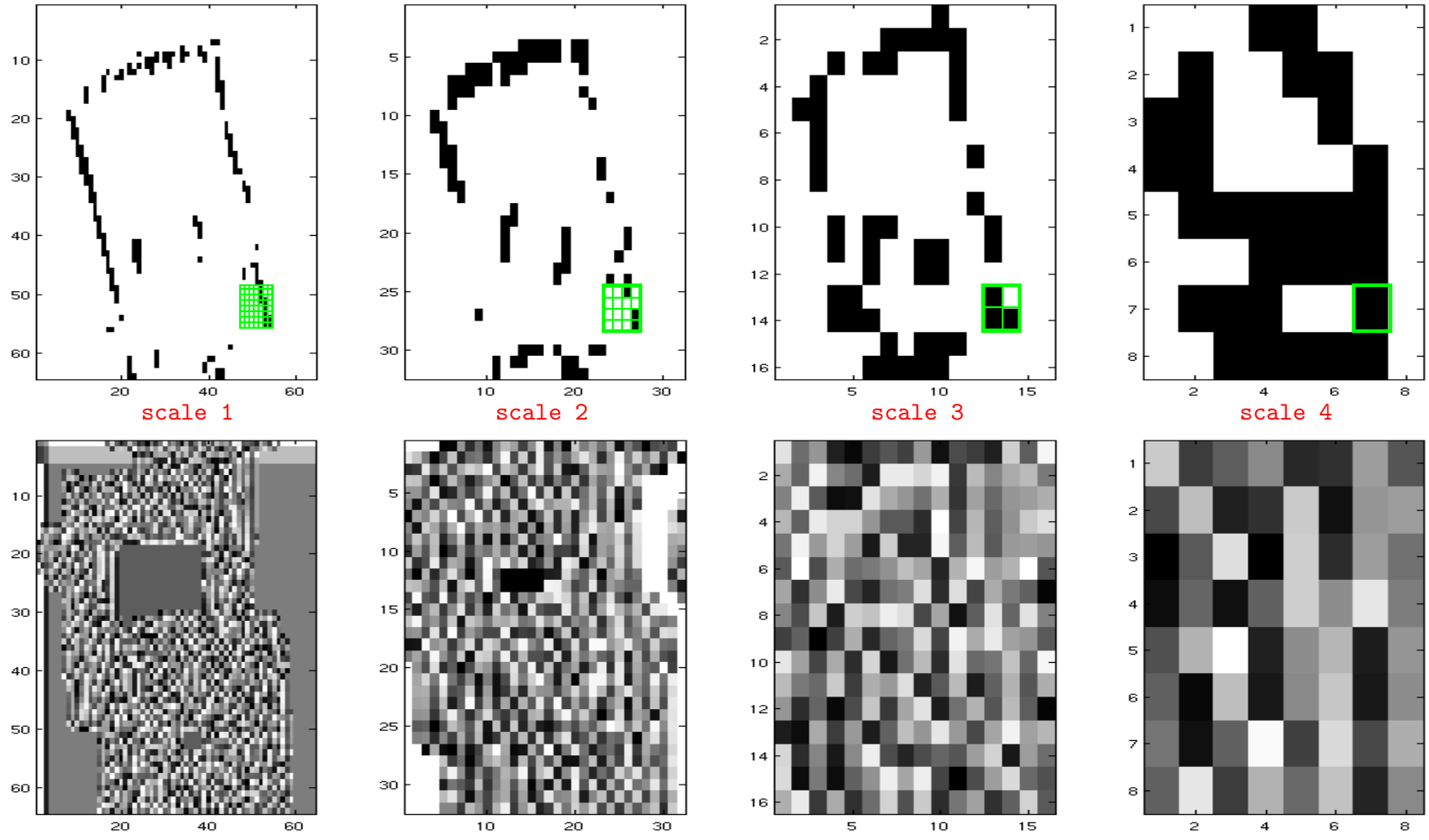

angle $\angle w_k$

Edge detection

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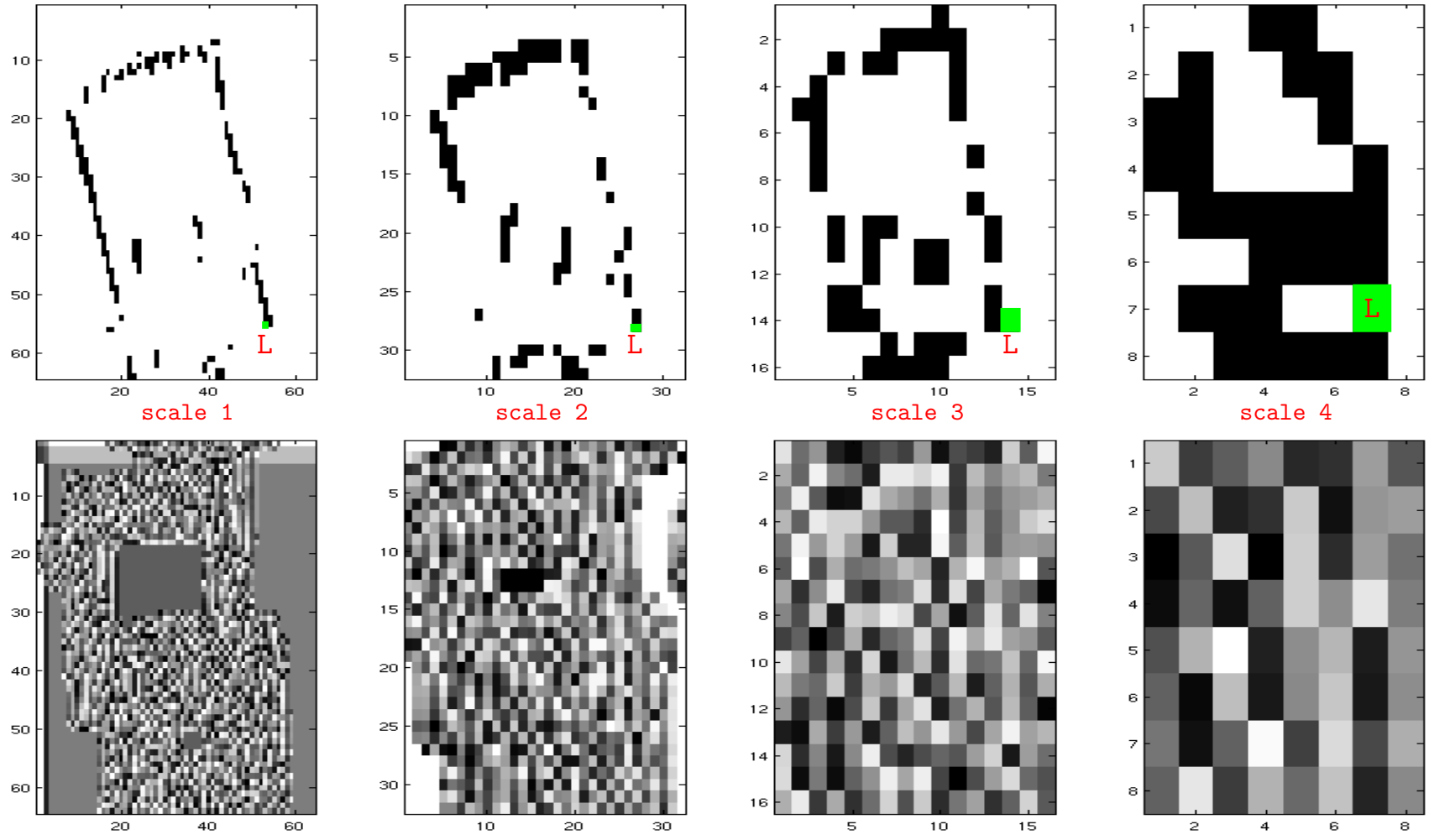
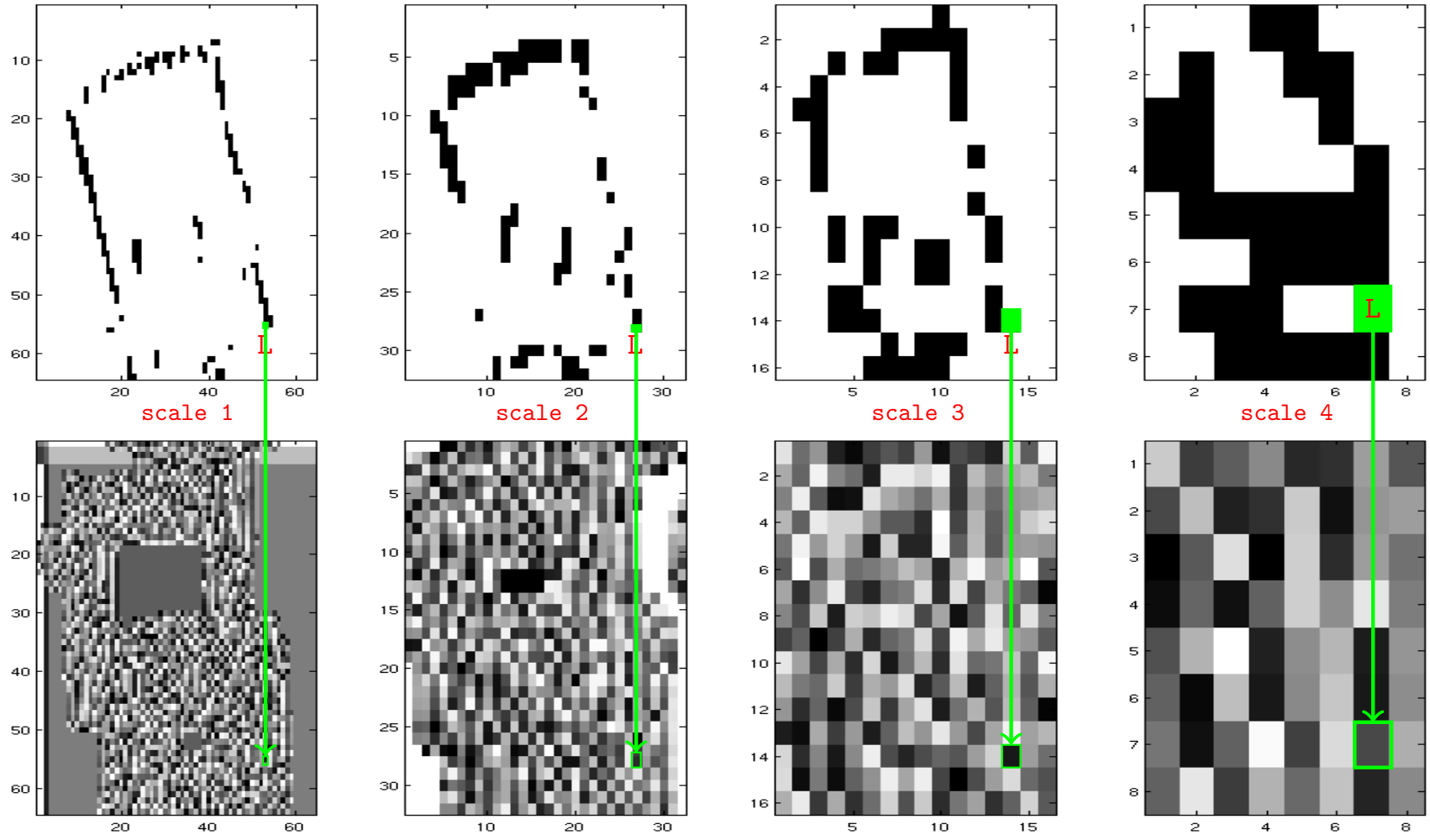


angle $\angle w_k$

Edge detection

S/L-state (S = white, L = black)



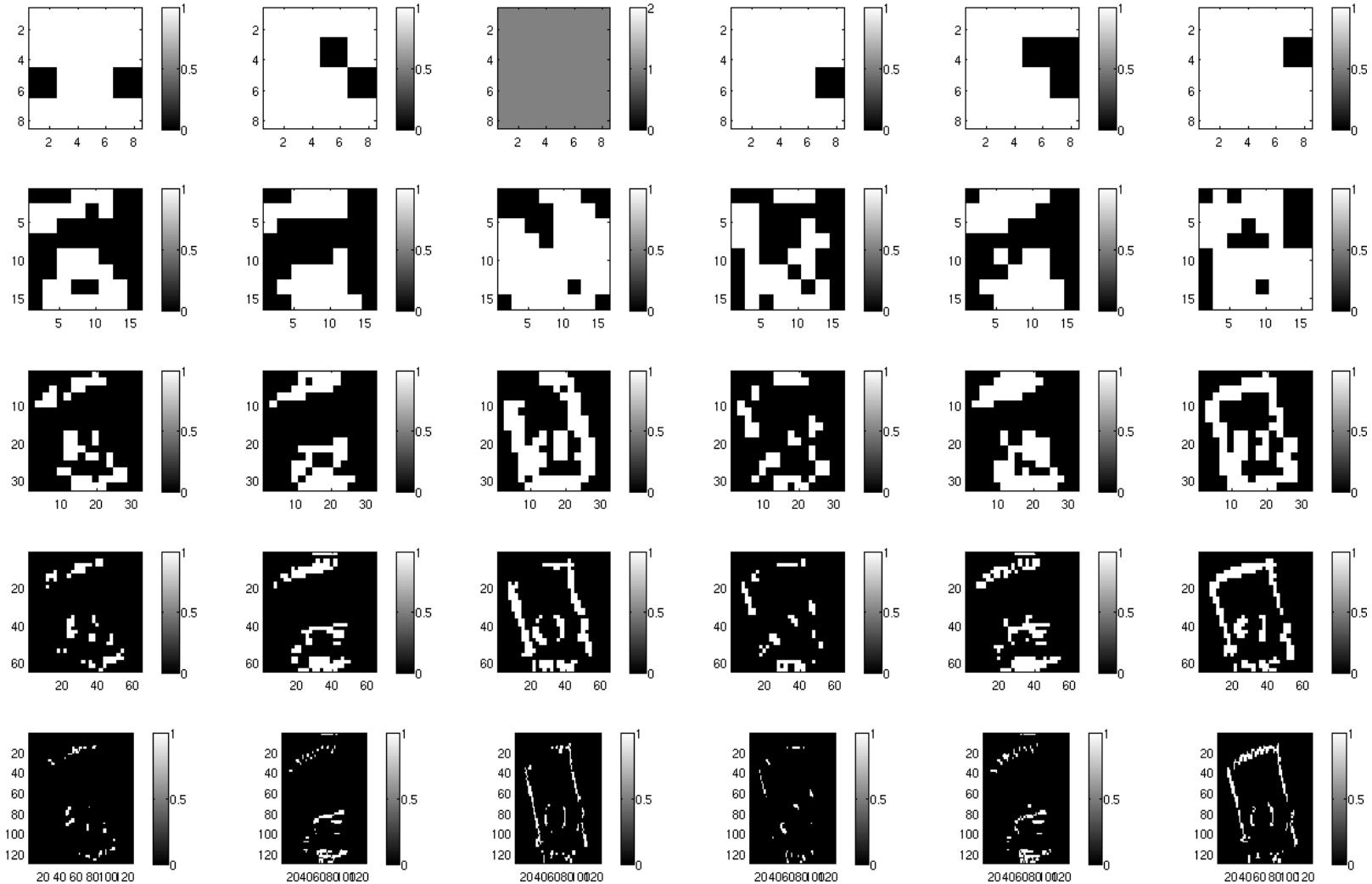
angle $\angle w_k$

Image Analysis

- Smooth Region. Both the coefficient and its parent have small magnitude (state `S`).
- Isolated Edge. Both coefficient and parent have large magnitude (state `L`), corresponding phases are statistically similar.
- Texture Region. Both coefficient and parent have large magnitude, but corresponding phases are very different.

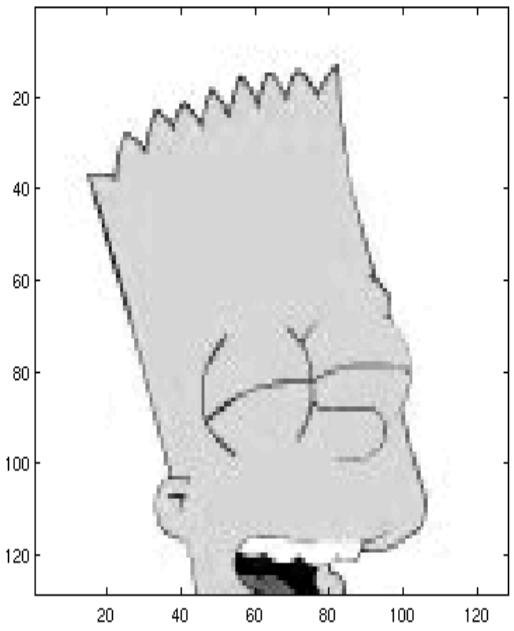
Experiments

Detection of sequences L-L-$\cdots$ with constant variance $\sigma = 0.5$

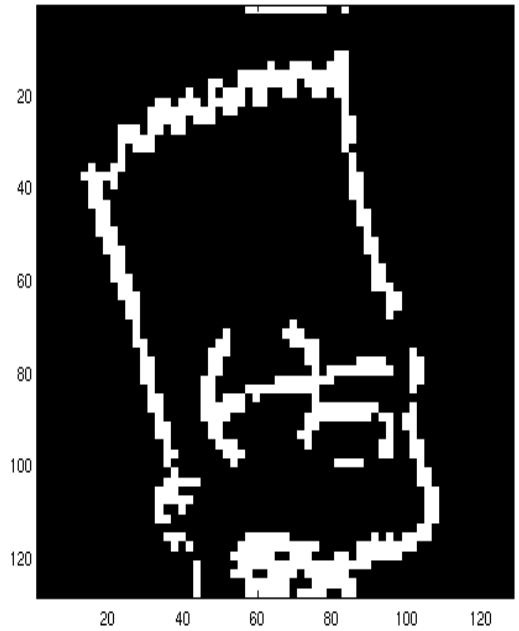


Experiments

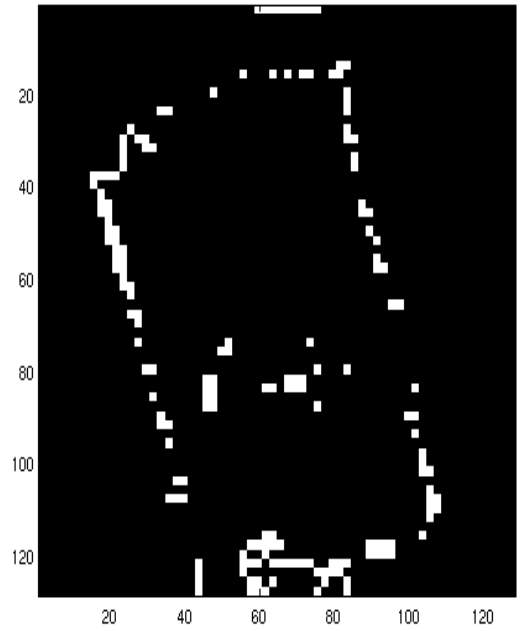
The results depend very strongly on variance



original



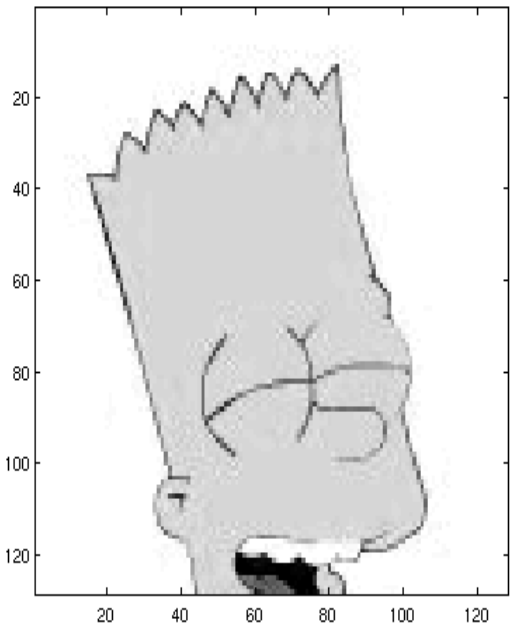
L-L-$\cdots$ map
($\sigma = 0.5$)



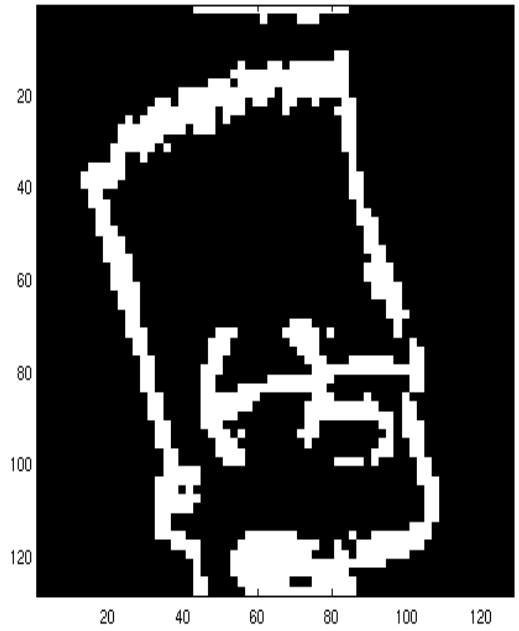
L-L-$\cdots$ map
($\sigma = 1$)

Experiments

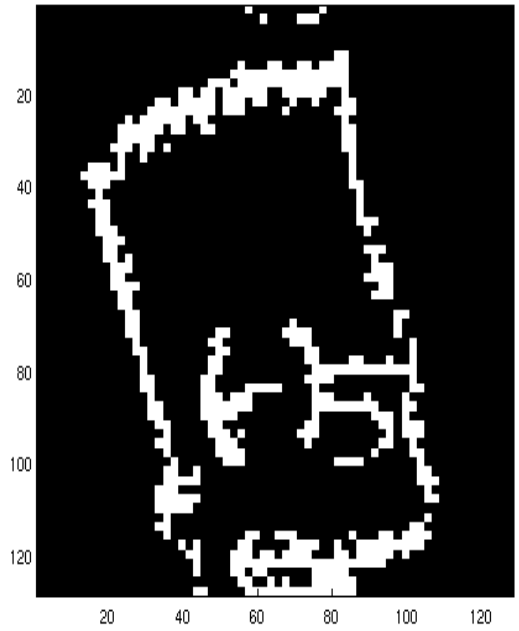
Removal of texture region ($\sigma = 0.4$)



original



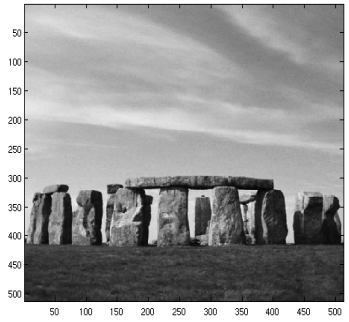
L-L-$\cdots$ map



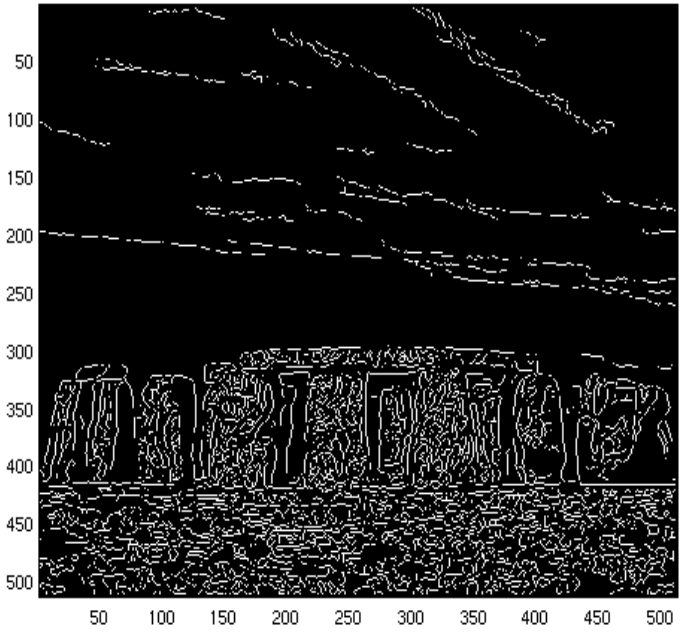
edges
(threshold $= 2$)

Experiments

Comparison to Canny

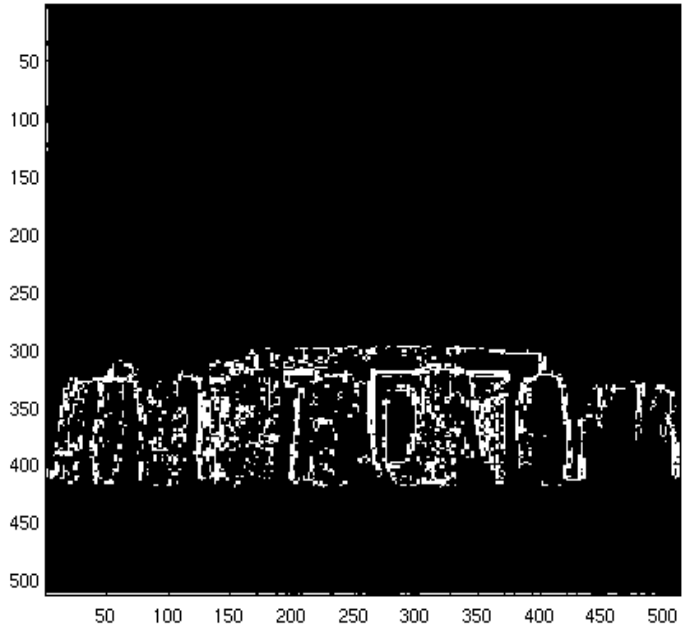


original



edge map (Canny)

`edge(IMG,'canny')`



edge map (HMM)

($\sigma = 0.3$)