Formulas for the Final Exam in MATH 142

springs $F(x) = k(x - x_0)$

$$W = \int_a^b F(x)\ dx$$

weights $F(x) = (Cx + M)g$

probability formulas:

exponential density $f(t) = \begin{cases} 0 & \textbf{if}\ \ t < 0 \\ ce^{-ct} & \textbf{if}\ \ t \geq 0 \end{cases}$ normal distribution $f(t) = \dfrac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

cylinders $V = \displaystyle\int_\alpha^\beta 2\pi\ R(u)\ \ (f(u) - g(u))\ \ du$

washers $V = \displaystyle\int_\alpha^\beta \pi\ \ \left(|f(u)|^2 - |g(u)|^2\right)\ \ du$

arclength $L = \displaystyle\int_\alpha^\beta \sqrt{|x'(t)|^2 + |y'(t)|^2}\ dt$ $\qquad L = \displaystyle\int_\alpha^\beta \sqrt{|r(\theta)|^2 + |r'(\theta)|^2}\ d\theta$

surface area in polar coordinates $A = \displaystyle\int_\alpha^\beta \frac{1}{2}\ \ \left(|f(\theta)|^2 - |g(\theta)|^2\right)\ \ d\theta$

surface area by a rotated curve $A = \displaystyle\int_\alpha^\beta 2\pi\ R(t)\ \sqrt{|x'(t)|^2 + |y'(t)|^2}\ dt$

power series:

$$f(x) = \sum_{k=0}^\infty \frac{f^{(k)}(x)}{k!}\ (x-a)^k$$

$$\tan^{-1} x = \sum_{k=0}^\infty \frac{(-1)^k}{2k+1}\ x^{2k+1} \qquad\qquad -1 \leq x \leq 1$$

$$(1+x)^m = 1 + \sum_{k=1}^\infty \frac{m(m-1)...(m-k+1)}{k!}\ x^k \qquad\qquad -1 < x < 1 \quad (m \neq 0, 1, 2, ...)$$

integrals:

$$\int f(x)g'(x)v\ dx = \int f(x)\ dg(x) = f(x)g(x) - \int g(x)\ df(x) = f(x)g(x) - \int g(x)f'(x)\ dx$$

$$\int u\ dv = uv - \int v\ du \qquad\qquad \int \frac{1}{x+a} = \ln(x+a) + C$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a}\ +\ C \qquad\qquad \int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|\ +\ C$$

$$\int \frac{du}{\sqrt{u^2+b}} = \ln\left|u + \sqrt{u^2+b}\right|\ +\ C \qquad\qquad \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a}\ +\ C$$

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$$e := \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \qquad \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = e^{-1}$$

$$\frac{dy}{dx} = \frac{r\cos\theta + \dfrac{dr}{d\theta}\sin\theta}{-r\sin\theta + \dfrac{dr}{d\theta}\cos\theta} \quad \text{for} \quad r = r(\theta)$$

trigonometric substitutions

$$\sqrt{a^2-x^2} \quad \to \quad x = a\sin t \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2} \quad \to \quad a^2-x^2 = a^2-a^2\sin^2 t = a^2\cos^2 t$$

$$\sqrt{a^2+x^2} \quad \to \quad x = a\tan t \quad -\frac{\pi}{2} < t < \frac{\pi}{2} \quad \to \quad a^2+x^2 = a^2+a^2\tan^2 t = a^2\sec^2 t$$

$$\sqrt{x^2-a^2} \quad \to \quad x = a\sec t \quad \begin{matrix} 0 \le t < \pi/2 & \textbf{if} & x \ge a > 0 \\ \pi/2 < t \le \pi & \textbf{if} & x \le a < 0 \end{matrix} \quad \to \quad x^2-a^2 = a^2\sec^2 t - a^2 = a^2\tan^2 t$$

$$\int \tan^m x\ \sec^n x\ dx \quad \rightrightarrows \quad \begin{cases} n \ \textbf{ even} & \to \quad u = \tan x \quad \to \quad \sec^2 x = \tan^2 x + 1 \\ m \ \textbf{ odd} & \to \quad u = \sec x \quad \to \quad \tan^2 x = \sec^2 x - 1 \\ m \ \textbf{ even} \ \text{ and } \ n \ \textbf{ odd} & \to \quad \tan^2 x = \sec^2 x - 1 \end{cases}$$

$$\int \sin^m x\ \cos^n x\ dx \quad \rightrightarrows \quad \begin{cases} n \ \textbf{ odd} & \to \quad u = \sin x \quad \to \quad \cos^2 x = 1 - \sin^2 x \\ m \ \textbf{ odd} & \to \quad u = \cos x \quad \to \quad \sin^2 x = 1 - \cos^2 x \\ m \ \textbf{ even} \ \text{ and } \ n \ \textbf{ even} & \to \quad \begin{cases} \sin^2 x = \frac{1}{2}(1-\cos 2x) \\ \cos^2 x = \frac{1}{2}(1+\cos 2x) \end{cases} \end{cases}$$

$$\sin\alpha\ \cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right] \qquad \cos^2 t = \frac{1}{2}(1+\cos 2t)$$

$$\sin\alpha\ \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \qquad \sin^2 t = \frac{1}{2}(1-\cos 2t)$$

$$\cos\alpha\ \cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right] \qquad \sin^2 t + \cos^2 t = 1$$

$$\sin(\alpha+\beta) = \sin\alpha\ \cos\beta + \cos\alpha\ \sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\ \cos\beta - \cos\alpha\ \sin\beta \qquad \sin 2t = 2\sin t\cos t$$

$$\cos(\alpha+\beta) = \cos\alpha\ \cos\beta + \sin\alpha\ \sin\beta \qquad \cos 2t = cos^2 t - \sin^2 t$$

$$\cos(\alpha-\beta) = \cos\alpha\ \cos\beta - \sin\alpha\ \sin\beta \qquad \cos 2t = 1 - 2\sin t = 2\cos t - 1$$

$$\int \sin^n x\ dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\ dx$$

$$\int \cos^n x\ dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\ dx$$