

Quiz #9

SOLUTION

{ 20 points } Find the length of the curve

$$x = t^3, \quad y = 2t^2 \quad (0 \leq t \leq 1)$$

$$\begin{aligned} L &= \int_0^1 \sqrt{(x')^2 + (y')^2} dt = \int_0^1 \sqrt{(3t^2)^2 + (4t)^2} dt = \int_0^1 \sqrt{9t^4 + 16t^2} dt \\ &= \int_0^1 |t| \sqrt{9t^2 + 16} dt = \frac{1}{18} \int_0^1 18t \sqrt{9t^2 + 16} dt \end{aligned}$$

Change of variables : $u = 9t^2 + 16$

$$\begin{aligned} du &= 18t dt & t = 0 &\rightarrow u = 16 \\ & & t = 1 &\rightarrow u = 25 \end{aligned}$$

$$\begin{aligned} L &= \frac{1}{18} \int_{16}^{25} \sqrt{u} du = \frac{1}{18} \left(\frac{2}{3} u^{\frac{3}{2}} \right) \Big|_{16}^{25} = \frac{1}{27} \left(25^{\frac{3}{2}} - 16^{\frac{3}{2}} \right) \\ &= \frac{125 - 64}{27} = \frac{61}{27} \end{aligned}$$