MATH 142 – Final Exam

$$\sqrt{a^2-x^2} \quad \rightarrow \quad x = a\sin t \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2} \quad \rightarrow \quad a^2 - x^2 = a^2 - a^2\sin^2 t = a^2\cos^2 t$$

$$\sqrt{a^2+x^2} \quad \rightarrow \quad x = a\tan t \quad -\frac{\pi}{2} < t < \frac{\pi}{2} \quad \rightarrow \quad a^2 + x^2 = a^2 + a^2\tan^2 t = a^2\sec^2 t$$

$$\sqrt{x^2-a^2} \quad \rightarrow \quad x = a\sec t \quad \begin{cases} 0 \le t < \pi/2 & \textbf{if } \ x \ge a \\ \pi/2 < t \le \pi & \textbf{if } \ x \le a \end{cases} \quad \rightarrow \quad x^2 - a^2 = a^2\sec^2 t - a^2 = a^2\tan^2 t$$

$$\int \tan^m x \ \sec^n x \ dx \quad \rightrightarrows \quad \begin{cases} n \ \ \textbf{even} & \rightarrow \quad u = \tan x \quad \rightarrow \quad \sec^2 x = \tan^2 x + 1 \\ m \ \ \textbf{odd} & \rightarrow \quad u = \sec x \quad \rightarrow \quad \tan^2 x = \sec^2 x - 1 \\ m \ \ \textbf{even} \ \text{ and } \ n \ \ \textbf{odd} & \rightarrow \quad \tan^2 x = \sec^2 x - 1 \end{cases}$$

$$\int \sin^m x \ \cos^n x \ dx \quad \rightrightarrows \quad \begin{cases} n \ \ \textbf{odd} & \rightarrow \quad u = \sin x \quad \rightarrow \quad \cos^2 x = 1 - \sin^2 x \\ m \ \ \textbf{odd} & \rightarrow \quad u = \cos x \quad \rightarrow \quad \sin^2 x = 1 - \cos^2 x \\ m \ \ \textbf{even} \ \text{ and } \ n \ \ \textbf{even} & \rightarrow \quad \begin{cases} \sin^2 x = \frac{1}{2}(1 - \cos 2x) \\ \cos^2 x = \frac{1}{2}(1 + \cos 2x) \end{cases} \end{cases}$$

$$\sin\alpha \ \cos\beta = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$$

$$\sin\alpha \ \sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$\cos\alpha \ \cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\sin(\alpha + \beta) = \sin\alpha \ \cos\beta \ + \ \cos\alpha \ \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \ \cos\beta \ - \ \cos\alpha \ \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \ \cos\beta \ + \ \sin\alpha \ \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \ \cos\beta \ - \ \sin\alpha \ \sin\beta$$

$$\sin(2\theta) = 2 \ \sin\theta \ \cos\theta \qquad \sin^2\theta \ + \ \cos^2\theta \ = \ 1$$

$$\int \sin^n x \ dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x \ dx$$

$$\int \cos^n x \ dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x \ dx$$

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$$\left|\int_a^b f(x)dx - M_n\right| \le \frac{(b-a)^3 K_2}{24n^2} \qquad \left|\int_a^b f(x)dx - T_n\right| \le \frac{(b-a)^3 K_2}{12n^2} \qquad \left|\int_a^b f(x)dx - S_n\right| \le \frac{(b-a)^5 K_4}{180(2n)^4}$$

$$\int u\ dv = uv - \int v\ du$$

$$\int_a^b f(x)\ dx \ = \ \lim_{q \to b^-} \int_a^q f(x)\ dx$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} \ + \ C$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| \ + \ C$$

$$\int \frac{du}{\sqrt{u^2+b}} = \ln\left|u+\sqrt{u^2+b}\right| \ + \ C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} \ + \ C$$

$$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\ x^{2k+1} \qquad -1 \le x \le 1$$

$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \frac{m(m-1)...(m-k+1)}{k!}\ x^k \qquad -1 < x < 1 \quad (m \ne 0, 1, 2, ...)$$

$$r = r(\theta)\ : \qquad \frac{dy}{dx} = \frac{r\cos\theta \ + \ \frac{dr}{d\theta}\ \sin\theta}{-r\sin\theta \ + \ \frac{dr}{d\theta}\ \cos\theta}$$