

## Quiz #6

### SOLUTION

1. { 12 points } Evaluate the integral

$$\int_1^e \sqrt{x} \ln x \, dx$$

$$\int_1^e \sqrt{x} \ln x \, dx = \int_1^e \ln x \, d\left(\frac{2}{3}x^{\frac{3}{2}}\right) = \frac{2}{3}x^{\frac{3}{2}} \ln x \Big|_1^e - \int_1^e \frac{2}{3}x^{\frac{3}{2}} \cdot \frac{1}{x} \, dx$$

$$= \frac{2}{3} \left( e^{\frac{3}{2}} \ln e - 1^{\frac{3}{2}} \ln 1 \right) - \frac{2}{3} \int_1^e x^{\frac{1}{2}} \, dx = \frac{2}{3} \left( e^{\frac{3}{2}} - 0 \right) - \frac{2}{3} \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_1^e$$

$$= \frac{2}{3} e^{\frac{3}{2}} - \frac{4}{9} \left( e^{\frac{3}{2}} - 1 \right) = \left( \frac{2}{3} - \frac{4}{9} \right) e^{\frac{3}{2}} + \frac{4}{9} = \frac{2}{9} e^{\frac{3}{2}} + \frac{4}{9}$$