Quiz #5

SOLUTION

1. { 12 points } **Find the average value of the function**

$$f(x) = \frac{e^{2x}}{1+e^{4x}}$$

over the interval $\left[0, \frac{\ln 3}{4}\right]$.

$$f_{ave} = \frac{1}{\frac{\ln 3}{4} - 0} \int_0^{\frac{\ln 3}{4}} \frac{e^{2x}}{1+e^{4x}}\, dx$$

Change of variables : $u = e^{2x}$

$$du = 2e^{2x}\, dx \qquad \begin{array}{lcl} x = 0 & \rightarrow & u = e^0 = 1 \\ t = \frac{\ln 3}{4} & \rightarrow & u = e^{\frac{\ln 3}{2}} = e^{\ln\sqrt{3}} = \sqrt{3} \end{array}$$

$$f_{ave} = \frac{4}{\ln 3} \cdot \frac{1}{2} \int_0^{\frac{\ln 3}{4}} \frac{2e^{2x}}{1+e^{4x}}\, dx = \frac{2}{\ln 3} \int_1^{\sqrt{3}} \frac{du}{1-u^2}$$

$$= \frac{2}{\ln 3} \tan^{-1} u \Big|_1^{\sqrt{3}} = \frac{2}{\ln 3}\left(\tan^{-1}\sqrt{3} - \tan^{-1} 1\right)$$

$$= \frac{2}{\ln 3}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{6\ln 3}$$