Quiz #4

SOLUTION

1. { 12 points } **Find the exact arc length of the parametric curve without eliminating the parameter**

$$x = 2t^2 \ , \qquad y = t^3 \qquad (0 \le t \le 1)$$

$$L \ = \ \int_0^1 \sqrt{(x')^2 + (y')^2} \ dt \ = \ \int_0^1 \sqrt{(4t)^2 + (3t^2)^2} \ dt \ = \ \int_0^1 \sqrt{16t^2 + 9t^4} \ dt$$

$$= \ \int_0^1 |t| \ \sqrt{16 + 9t^2} \ dt \ = \ \frac{1}{18} \int_0^1 18t \ \sqrt{16 + 9t^2} \ dt$$

Change of variables : $\quad u = 16 + 9t^2$

$$du \ = \ 18t \ dt \qquad\qquad \begin{array}{lcl} t = 0 & \rightarrow & u = 16 \\ t = 1 & \rightarrow & u = 25 \end{array}$$

$$L \ = \ \frac{1}{18} \int_{16}^{25} \sqrt{u} \ du \ = \ \frac{1}{18} \left(\frac{2}{3} u^{\frac{3}{2}} \right) \Big|_{16}^{25} \ = \ \frac{1}{27} \left(25^{\frac{3}{2}} - 16^{\frac{3}{2}} \right)$$

$$= \ \frac{125 - 64}{27} \ = \ \frac{61}{27}$$