Quiz #1

SOLUTION

1. { 12 points } **Evaluate the integral**

$$\int_{-1}^{3} \sqrt{1+|x|}\, dx$$

$$= \int_{-1}^{0} \sqrt{1+|x|}\, dx \; + \int_{0}^{3} \sqrt{1+|x|}\, dx \; = \; \int_{-1}^{0} \sqrt{1-x}\, dx \; + \int_{0}^{3} \sqrt{1+x}\, dx$$

$$= -\int_{-1}^{0} (1-x)^{\frac{1}{2}}\, d(1-x) \; + \; \int_{0}^{3} (1+x)^{\frac{1}{2}}\, d(1+x)$$

$$= -\frac{2}{3}\,(1-x)^{\frac{3}{2}}\Big|_{-1}^{0} \; + \; \frac{2}{3}\,(1+x)^{\frac{3}{2}}\Big|_{0}^{3}$$

$$= -\frac{2}{3}\left((1-0)^{\frac{3}{2}} - (1-(-1))^{\frac{3}{2}}\right) \; + \; \frac{2}{3}\left((1+3)^{\frac{3}{2}} - (1+0)^{\frac{3}{2}}\right)$$

$$= -\frac{2}{3}\left(1 - 2^{\frac{3}{2}}\right) \; + \; \frac{2}{3}\left(4^{\frac{3}{2}} - 1\right)$$

$$= -\frac{2}{3} + \frac{2}{3}(2\sqrt{2}) \; + \; \frac{2}{3}(8-1) \; = \; -\frac{2}{3} + \frac{4}{3}\sqrt{2} \; + \; \frac{14}{3}$$

$$= 4 + \frac{4}{3}\sqrt{2}$$