Quiz #15

SOLUTION

1. { 12 points } **Find the radius of convergence and the interval of convergence.**

$$\sum_{k=1}^{\infty}(-1)^k \frac{(x-3)^k}{k^2+1}$$

$$\frac{a_{k+1}}{a_k} = \frac{(-1)^{k+1}(x-3)^{k+1} \;\cdot\; \left(k^2+1\right)^k}{\left((k+1)^2+1\right)^k \;\cdot\; (-1)^k(x-3)^k} = \frac{(-1)(x-3)(k^2+1)}{k^2+2k+2} \to \; -(x-3)$$

Therefore the series is absolutely convergent for $|x-3| < 1$ **and divergent for** $|x-3| > 1$**, i.e. the radius of convergence is** 1**. To determine the convergence for** $|x-3| = 1$ **we observe that then** $|a_k| = \dfrac{1}{k^2+1}$ **and**

$$\lim_{k\to\infty} \frac{|a_k|}{k^{-2}} = 1$$

Thus, the series for $|x-3| = 1$ **is absolutely convergent together with the** p**-series** $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ **for** $p = 2 > 1$ **which is convergent. That gives the interval of convergence** $2 \leq x \leq 4 \;\Leftrightarrow\; |x-3| \leq 1$**.**