

Quiz #14

SOLUTION

1. { 12 points } Find the Maclaurin polynomials of orders $n = 0, 1, 2, 3$, and then find the n th Maclaurin polynomial for the function xe^x in sigma notation.

$$\begin{aligned} f(x) &= xe^x & f(0) &= 0 \\ f'(x) &= xe^x + e^x & f'(0) &= 1 \\ f''(x) &= xe^x + 2e^x & f''(0) &= 2 \\ f'''(x) &= xe^x + 3e^x & f'''(0) &= 3 \\ f^{(k)}(x) &= xe^x + ke^x & f^{(k)}(0) &= k \end{aligned}$$

$$M_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k = \sum_{k=1}^n \frac{k}{k!} x^k = \sum_{k=1}^n \frac{1}{(k-1)!} x^k$$

In particular,

$$M_0(x) = 0, \quad M_1(x) = x, \quad M_2(x) = x^2 + x, \quad M_3(x) = \frac{1}{2}x^3 + x^2 + x.$$