Quiz #13

SOLUTION

1. { 12 points } **Use any method to determine whether the series converges or diverges.**

(A) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{(k-1)k(k+1)}}$

$$\frac{a_k}{k^{-\frac{3}{2}}} = \frac{\sqrt{k^3}}{\sqrt{(k-1)k(k+1)}} \to 1$$

and therefore the series is *convergent* together with the p-series for $p = \frac{3}{2} > 1$.

(B) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{\ln k}\right)^k$

$$a_k^{\frac{1}{k}} = \left(\frac{1}{\ln k}\right)^{\frac{k}{k}} = \frac{1}{\ln k} \to 0 < 1$$

and therefore the series is *convergent* by the root test.

(C) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k\ k^3}{k^2+1}$

$$\lim_{k\to\infty} \frac{(-1)^k\ k^3}{k^2+1} \neq 0$$

and therefore the series is *divergent*.

(D) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{\frac{1}{3}}}$

$\{a_k\} = \left\{\frac{1}{k^{\frac{1}{3}}}\right\}$ **is a decreasing sequence and** $\displaystyle\lim_{k\to\infty} \frac{1}{k^{\frac{1}{3}}} = 0$. **Thus, the alternating series *converges*.**