Quiz #11

SOLUTION

1. { 12 points } **Determine whether the series converges or diverges. Explain why.**

(A) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{1}{2^k}-\frac{1}{2^{k-1}}\right)$

$$S_n=\sum_{k=1}^{n}\left(\frac{1}{2^k}-\frac{1}{2^{k-1}}\right)=\frac{1}{2^n}-\frac{1}{2^0}=\frac{1}{2^n}-1\ \rightarrow\ -1$$

Therefore the series is convergent and its sum is -1.

(B) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{4}{3}\right)^k$

The general term $u_k=\left(\frac{4}{3}\right)^k$ does not have limit 0 as $k\to\infty$. Thus, the series is divergent.

(C) $\displaystyle\sum_{k=1}^{\infty}k^{-\frac{5}{2}}$

This is a p-series with $p=\frac{5}{2}>1$. Thus, it converges.

(D) $\displaystyle\sum_{k=1}^{\infty}\frac{3k^2}{2+k^3}$

The integral test is applicable since the function $f(x)=\dfrac{3x^2}{2+x^3}$ is decreasing for large x (e.g. for $x>2$) and tends to 0 as $x\to\infty$.

$$\int_2^{\infty}\frac{3x^2}{2+x^3}dx=\int_2^{\infty}\frac{d(2+x^3)}{2+x^3}=\ln|2+x^3|\Big|_2^{\infty}$$

Since $\lim\limits_{x\to\infty}\ln|2+x^3|=\infty$, the integral is divergent. This implies that the series also diverges.