Quiz #11

SOLUTION

1. { 12 points } **Show that the given sequence is strictly increasing or strictly decreasing. If it is neither, state whether the sequence is eventually strictly increasing or eventually strictly decreasing.**

(A) $\left\{ \dfrac{3^n}{n!} \right\}_{n=2}^{\infty}$

$$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}\quad n!}{(n+1)!\quad 3^n} = \frac{3}{n+1} < 1 \quad \textbf{for} \quad n > 2$$

The sequence is decreasing and eventually strictly decreasing.

(B) $\left\{ n^4 e^{-n} \right\}_{n=1}^{\infty}$

$$f(x) := x^4 e^{-x} \qquad f'(x) = 4x^3 e^{-x} - x^4 e^{-x} = x^3(4-x)e^{-x} < 0 \quad \textbf{for} \quad x > 4$$

Thus, $f(n+1) < f(n)$ **for** $n > 4$ **and the sequence is eventually strictly decreasing.**

(C) $\left\{ \dfrac{1}{n - \ln n} \right\}_{n=1}^{\infty}$

$$\frac{a_{n+1}}{a_n} = \frac{n - \ln n}{n + 1 - \ln(n+1)} < 1 \quad \textbf{if} \quad n - \ln n < n + 1 - \ln(n+1)$$

$$\Leftrightarrow \quad \ln(n+1) < 1 + \ln n = \ln e + \ln n = \ln(en) \quad \Leftrightarrow \quad n + 1 < en$$

Since $\dfrac{n+1}{n} < e$ **for** $n \geq 1$, **the sequence is strictly decreasing.**