Solutions for Quiz #9

October 23, 2003

1. { 2 points } **How many functions are there from a set with five elements to a set with two elements?**

$$2^5 = 32 \ .$$

2. { 2 points } **Find a function** f **defined on a set of nonnegative integers that defines a sequence whose first six terms are**

$$0, \quad -2, \ 4, \quad -6, \ 8, \quad -10$$

example: $\qquad f(n) = (-1)^n(2n) \ .$

3. { 2 points } **Fill in the blank**

$$\log_3 \tfrac{1}{9} = -2 \quad \text{because} \quad \tfrac{1}{9} = 3^{-2} \quad .$$

4. **Let** $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$, **and** $Z = \{1, 2\}$.

(A) { 2 points } **Define a function** $f : X \to Z$ **that is onto but not one-to-one.**

example: $\qquad f(1) = 1, \ \ f(2) = 2, \ \ f(3) = 1.$

(B) { 2 points } **Define a function** $g : X \to Y$ **that is neither one-to-one nor onto.**

example: $\qquad g(1) = 1, \ \ g(2) = 2, \ \ g(3) = 1.$

Bonus. { 5 points } **Determine whether or not the function** $f(x) = \frac{x-1}{x+1}$ **, defined for all real numbers** $x \neq -1$ **, is one-to-one and justify your answer.**

Let $f(x) = f(y)$ for some arbitrary real numbers $x, y \neq -1$. Then

$$\frac{x-1}{x+1} = \frac{y-1}{y+1}$$

and therefore

$(x-1)(y+1) = (y-1)(x+1)$, $\quad$ i.e. $\qquad xy + x - y - 1 = xy + y - x - 1$.

That gives

$$2x = 2y$$

which proves that if $f(x) = f(y)$ then $x = y$, and therefore f is one-to-one.