Solutions for Quiz #8

October 21, 2003

1. Suppose that three computer boards in a production run of forty are defective. A sample of four is to be selected to be checked for defects.

(A) { 2 points } **How many different samples can be chosen?**

$$A = \binom{40}{4} = \frac{40!}{36!\ 4!} = \frac{40 \cdot 39 \cdot 38 \cdot 37}{4 \cdot 3 \cdot 2 \cdot 1} = 91\ 390\ .$$

(B) { 2 points } **How many samples will contain at least one defective board?**

The samples that contain no defective boards can be considered as samples of four out of $40 - 3 = 37$ elements. So their number is $\binom{37}{4}$. Thus,

$$B = \binom{40}{4} - \binom{37}{4} = \frac{(40 \cdot 39 \cdot 38 \cdot 37) - (37 \cdot 36 \cdot 35 \cdot 34)}{4 \cdot 3 \cdot 2 \cdot 1} = 91\ 390 - 66\ 045 = 25\ 345\ .$$

(C) { 2 points } **What is the probability that a randomly chosen sample of four will contain at least one defective board?**

The probability is the quotient of the numbers from **(B)** and **(A)**

$$p = \frac{B}{A} = \frac{25\ 345}{91\ 390} = \frac{137}{494} \approx 0.277\ .$$

2. { 3 points } **Using the definition of** $\binom{n}{r}$ **show that for all integers** n **and** k **with** $r + 1 \le n$

$$\binom{n}{r+1} = \frac{n-r}{r+1} \binom{n}{r}\ .$$

$$\binom{n}{r+1} = \frac{n!}{(n-r-1)!\ (r+1)!} = \frac{n!\ (n-r)}{[(n-r-1)!\ (n-r)] \cdot [r!\ (r+1)]}$$
$$= \frac{n-r}{r+1} \cdot \frac{n!}{(n-r)!\ r!} = \frac{n-r}{r+1} \binom{n}{r}\ .$$

3. { 3 points } **Expand the expression using the binomial theorem.**

$$\left(\frac{2}{a} - \frac{a}{2}\right)^5$$

$$= \left(\frac{2}{a}\right)^5 - 5\left(\frac{2}{a}\right)^4 \left(\frac{a}{2}\right) + 10\left(\frac{2}{a}\right)^3 \left(\frac{a}{2}\right)^2 - 10\left(\frac{2}{a}\right)^2 \left(\frac{a}{2}\right)^3 + 5\left(\frac{2}{a}\right)\left(\frac{a}{2}\right)^4 - \left(\frac{a}{2}\right)^5$$
$$= \left(\frac{2}{a}\right)^5 - 5\left(\frac{2}{a}\right)^3 + 10\left(\frac{2}{a}\right) - 10\left(\frac{a}{2}\right) + 5\left(\frac{a}{2}\right)^3 - \left(\frac{a}{2}\right)^5\ .$$