

1. Suppose that three computer boards in a production run of forty are defective. A sample of four is to be selected to be checked for defects.

(A) { 2 points } How many different samples can be chosen?

$$A = \binom{40}{4} = \frac{40!}{36! 4!} = \frac{40 \cdot 39 \cdot 38 \cdot 37}{4 \cdot 3 \cdot 2 \cdot 1} = 91\,390.$$

(B) { 2 points } How many samples will contain at least one defective board?

The samples that contain no defective boards can be considered as samples of four out of  $40 - 3 = 37$  elements. So their number is  $\binom{37}{4}$ . Thus,

$$B = \binom{40}{4} - \binom{37}{4} = \frac{(40 \cdot 39 \cdot 38 \cdot 37) - (37 \cdot 36 \cdot 35 \cdot 34)}{4 \cdot 3 \cdot 2 \cdot 1} = 91\,390 - 66\,045 = 25\,345.$$

(C) { 2 points } What is the probability that a randomly chosen sample of four will contain at least one defective board?

The probability is the quotient of the numbers from (B) and (A)

$$p = \frac{B}{A} = \frac{25\,345}{91\,390} = \frac{137}{494} \approx 0.277.$$

2. { 3 points } Using the definition of  $\binom{n}{r}$  show that for all integers  $n$  and  $k$  with  $r + 1 \leq n$

$$\binom{n}{r+1} = \frac{n-r}{r+1} \binom{n}{r}.$$

$$\begin{aligned} \binom{n}{r+1} &= \frac{n!}{(n-r-1)! (r+1)!} = \frac{n! (n-r)}{[(n-r-1)! (n-r)] \cdot [r! (r+1)]} \\ &= \frac{n-r}{r+1} \cdot \frac{n!}{(n-r)! r!} = \frac{n-r}{r+1} \binom{n}{r}. \end{aligned}$$

3. { 3 points } Expand the expression using the binomial theorem.

$$\begin{aligned} &\left(\frac{2}{a} - \frac{a}{2}\right)^5 \\ &= \left(\frac{2}{a}\right)^5 - 5 \left(\frac{2}{a}\right)^4 \left(\frac{a}{2}\right) + 10 \left(\frac{2}{a}\right)^3 \left(\frac{a}{2}\right)^2 - 10 \left(\frac{2}{a}\right)^2 \left(\frac{a}{2}\right)^3 + 5 \left(\frac{2}{a}\right) \left(\frac{a}{2}\right)^4 - \left(\frac{a}{2}\right)^5 \\ &= \left(\frac{2}{a}\right)^5 - 5 \left(\frac{2}{a}\right)^3 + 10 \left(\frac{2}{a}\right) - 10 \left(\frac{a}{2}\right) + 5 \left(\frac{a}{2}\right)^3 - \left(\frac{a}{2}\right)^5. \end{aligned}$$