Solutions for Quiz #7 October 9, 2003

1. { 2 points } **If the largest of 87 consecutive integers is 326, what is the smallest?**

240 : $A_{87} = A_1 + 86$ and therefore $A_1 = 326 - 86 = 240$.

2. { 4 points } **A combination lock requires three selections of numbers, each from 1 to 20.**

(A) How many different combinations are possible?

$$20(20)(20) = 8000 .$$

(B) Suppose the lock is constructed in such a way that no number can be used twice. How many different combinations are possible?

$$20(19)(18) = 6840 .$$

3. { 4 points }

(A) How many 3-permutations are there of a set of five objects?

$$P(5,3) = 5(4)(3) = 60 .$$

(B) How many 2-permutations are there of a set of seven objects?

$$P(7,2) = 7(6) = 42 .$$

Bonus Problem. { 5 points } **How many integers from 1 to 1000 are neither multiples of 4 nor of 7 ?**

Let A be the set of the integers from 1 to 1000 that are multiple of 4, and B be the set of the integers from 1 to 1000 that are multiple of 7. It is easy to see that

$$n(A) = \left\lfloor \frac{1000}{4} \right\rfloor = 250 \qquad \text{and} \qquad n(B) = \left\lfloor \frac{1000}{7} \right\rfloor = 142 .$$

The set X of the integers from 1 to 1000 that are neither multiples of 4 nor of 7 can be represented as

$$X = A^c \cap B^c = (A \cup B)^c = U - (A \cup B) ,$$

where the universal set U consists of the integers from 1 to 1000. Thus,

$$n(X) = 1000 - n(A \cup B) .$$

To calculate $n(A \cup B)$ we shall apply the formula

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) .$$

Using that the set $A \cap B$ represents the numbers that are divisible both by 4 and 7, and therefore by 28, we receive

$$n(A \cap B) = \left\lfloor \frac{1000}{28} \right\rfloor = 35 .$$

Finally,

$$n(X) = 1000 - \{n(A) + n(B) - n(A \cap B)\} = 1000 - (250 + 142 - 35) = 1000 - 357 = 643 .$$