Solutions for Quiz #6 October 2, 2003

1. { 1 point for each correct answer } **For each of the following statements determine whether it is true or false.** *It is not necessary to justify your answer.*

(A) $\{1\} \subseteq \{1,2\}$. **true**

1 is the only element of the first set, and it belongs to the second set.

(B) $1 \in \{\{1\},2\}$. **false**

The elements of the set are $\{1\}$ and 2, but 1 is not among them since $1 \neq \{1\}$.

(C) For all sets A, B, and C, if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$. **true**

Let $x \in A$. If the condition $(A \subseteq B) \wedge (A \subseteq C)$ is satisfied, we have $x \in B$ and $x \in C$. Thus $x \in B \cap C$.

(D) For all sets A and B, if $A \subseteq B$ then $B^c \subseteq A^c$. **true**

The definition of $A \subseteq B$ is $\forall x, x \in A \rightarrow x \in B$. The contrapositive statement is $\forall x, x \notin B \rightarrow x \notin A$. The last is the definition of $B^c \subseteq A^c$.

(E) For all sets A and B, $(A \cup B)^c = A^c \cup B^c$. **false**

This is not the De Morgan's Law. The right identity is $(A \cup B)^c = A^c \cap B^c$.

(F) For all sets A, B, and C, $(A - B) \cup (A \cap B) = A$. **true**

See the solution of the Bonus Problem.

2. { 4 points } **Let** $A = \{b, c, d, f, g\}$ **and** $B = \{a, b, c\}$. **Find each of the following:**

(A) $A \cup B = \{a, b, c, d, e, f, g\}$.

(B) $A \cap B = \{b, c\}$.

(C) $A - B = \{d, f, g\}$.

(D) $B - A = \{a\}$.

Bonus Problem. { 5 points } **Explain your answer of 1 (F).**

This is the proof of the identity:

$$(A - B) \cup (A \cap B) = (A \cap B^c) \cup (A \cap B)$$

$$= A \cap (B^c \cup B) = A \cap U = A\,,$$

where U is the universal set.

We have used the definition of $A - B$, the distributive law, the definition of B^c, and $A \cap U = A$.