Solutions for Quiz #13

November 20, 2003

1. { 3 points } **Suppose a graph has vertices of degrees 1, 1, 4, 4, and 6. How many edges does the graph have?**

In the count of the degree of the graph every edge is counted twice. So, the number of edges is

$$\frac{1}{2}(1+1+4+4+6) = \frac{16}{2} = 8 \ .$$

2. { 4 points }
(A) In a group of 15 people, is it possible for each to shake hands with exactly 3 other persons? Explain.

It is not possible. The corresponding graph would have had 15 vertices each of degree 3 , i.e. a graph of degree $15 \cdot 3 = 45$ which is an odd number.

(B) In a group of 4 people, is it possible for each to shake hands with exactly 3 other persons? Explain.

It is possible. The corresponding graph is the **complete** graph with four vertices, i.e. each person shakes hands with the other three.

3. { 3 points } **Either draw** a graph with four vertices of degrees 1, 2, 3, 4 **or explain why no such graph exists.**

Here is an example for which one can draw a graph. Let the vertices are v_1 , v_2 , v_3 , and v_4 . Then the edges are defined as follows:
$e_1 = \{v_1, v_3\}$, $e_2 = \{v_2, v_3\}$, $e_3 = \{v_2, v_4\}$, $e_4 = \{v_3, v_4\}$, and $e_5 = \{v_4\}$.

Bonus. { 5 points } **Either draw** a simple graph with nine edges and all vertices of degree 3 **or explain why no such graph exists.**

Let n be the number of vertices of the graph. Then the degree of the graph is $3n$. But it can be calculated as twice the number of edges, as well. So, $3n = 2 \cdot 9 = 18$ and therefore $n = 6$.

Here is an example for which one can draw a graph. Let the vertices are v_1 , v_2 , v_3 , v_4 , v_5 , and v_6 . Then the edges are defined as follows:
$e_1 = \{v_1, v_2\}$, $e_2 = \{v_1, v_3\}$, $e_3 = \{v_1, v_4\}$, $e_4 = \{v_2, v_3\}$, $e_5 = \{v_2, v_5\}$, $e_6 = \{v_3, v_6\}$, $e_7 = \{v_4, v_5\}$, $e_8 = \{v_4, v_6\}$, and $e_1 = \{v_5, v_6\}$.