# Solutions for Quiz #13

November 20, 2003

**1.** { 3 points } **Suppose a graph has vertices of degrees 1, 1, 4, 4, and 6. How many edges does the graph have?**

In the count of the degree of the graph every edge is counted twice. So, the number of edges is

$$\frac{1}{2}(1+1+4+4+6) = \frac{16}{2} = 8 \ .$$

**2.** { 4 points }

**(A) In a group of 15 people, is it possible for each to shake hands with exactly 3 other persons? Explain.**

It is not possible. The corresponding graph would have had 15 vertices each of degree 3 , i.e. a graph of degree $15 \cdot 3 = 45$ which is an odd number.

**(B) In a group of 4 people, is it possible for each to shake hands with exactly 3 other persons? Explain.**

It is possible. The corresponding graph is the **complete** graph with four vertices, i.e. each person shakes hands with the other three.

**3.** { 3 points } **Either draw** a graph with four vertices of degrees 1, 2, 3, 4 **or explain why no such graph exists.**

**Here is an example for which one can draw a graph.** Let the vertices are $v_1$ , $v_2$ , $v_3$ , and $v_4$ . Then the edges are defined as follows:
$e_1 = \{v_1, v_3\}$ , $e_2 = \{v_2, v_3\}$ , $e_3 = \{v_2, v_4\}$ , $e_4 = \{v_3, v_4\}$ , and $e_5 = \{v_4\}$ .

**Bonus.** { 5 points } **Either draw** a simple graph with nine edges and all vertices of degree 3 **or explain why no such graph exists.**

Let $n$ be the number of vertices of the graph. Then the degree of the graph is $3n$. But it can be calculated as twice the number of edges, as well. So, $3n = 2 \cdot 9 = 18$ and therefore $n = 6$ .

**Here is an example for which one can draw a graph.** Let the vertices are $v_1$ , $v_2$ , $v_3$ , $v_4$ , $v_5$ , and $v_6$ . Then the edges are defined as follows:
$e_1 = \{v_1, v_2\}$ , $e_2 = \{v_1, v_3\}$ , $e_3 = \{v_1, v_4\}$ , $e_4 = \{v_2, v_3\}$ , $e_5 = \{v_2, v_5\}$ , $e_6 = \{v_3, v_6\}$ , $e_7 = \{v_4, v_5\}$ , $e_8 = \{v_4, v_6\}$ , and $e_1 = \{v_5, v_6\}$ .