Solutions for Quiz #12

November 18, 2003

1. { 3 points } **The relation** $\mathcal{R} = \{(a,a),(b,b),(b,d),(c,c),(d,b),(d,d)\}$ **is an equivalence relation on the set** $A = \{a,b,c,d\}$**. Find the distinct equivalence classes of** $\mathcal{R}$**.**

The distinct equivalence classes of $\mathcal{R}$ are given by the sets

$$\{a\}\ , \qquad \{b,d\}\ , \quad \text{and} \qquad \{c\}\ .$$

The following is the directed graph for $\mathcal{R}$.

2. { 4 points } **Let** $\mathcal{T}$ **be the relation of congruence modulo** 7**. Which of the following equivalence classes are equal ?**

$$[35],[3],[-7],[12],[0],[-2],[17]$$

Two equivalence classes $[m]$ and $[n]$ are equal if $m\,\mathcal{T}\,n$, i.e. if $7 \mid (m-n)$. Thus,

$$[35] = [-7] = [0]\ , \qquad [3] = [17]\ , \qquad [12] = [-2]\ .$$

3. { 3 points } **Let** A **be the set of all statement forms in three variables** p, q, r**.** $\mathcal{S}$ **is the relation defined as follows:**

for all P **and** Q **in** A **,**

$P\,\mathcal{S}\,Q \quad \Leftrightarrow \quad P$ **and** Q **have the same truth table.**

Describe distinct equivalence classes of $\mathcal{S}$**.**

The truth table for any statement forms in three has eight positions that correspond to the following triplets of True/False values for variables p, q, r:

$$TTT\ ,\quad TTF\ ,\quad TFT\ ,\quad TFF\ ,\quad FTT\ ,\quad FTF\ ,\quad FFT\ ,\quad FFF\ .$$

Each of these position has the value T or F . Thus, each equivalence class can be described by an ordered set of these eight T/F values.

Bonus. { 2 points } **How many are the classes from Problem 3?**

$$2^8 = 256$$