Solutions for Quiz #11

November 13, 2003

1. **Let** $C = \{2, 3, 4, 5\}$ **and** $D = \{3, 4\}$**. Define binary relation** $\mathcal{S}$ **from** C **to** D **as follows**

$$\text{for all } (x, y) \in C \times D, \ \ (x, y) \in \mathcal{S} \Leftrightarrow x \geq y.$$

(A) { 2 points } **Write** $\mathcal{S}$ **as a set of ordered pairs**

$$\mathcal{S} = \{(3, 3), (4, 3), (4, 4), (5, 3), (5, 4)\} .$$

(B) { 1 point for each correct answer } **For each of the following statements determine whether it is true or false.** *It is not necessary to justify your answer.*

$2 \ \mathcal{S} \ 4$	$4 \ \mathcal{S} \ 3$	$(2, 3) \in \mathcal{S}$
false	true	false

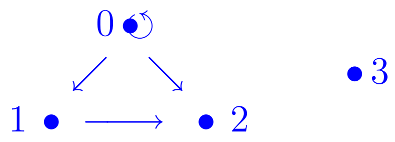
2. **The binary relation** $\mathcal{R}$ **is defined on the set** $A = \{0, 1, 2, 3\}$ **with**

$$\mathcal{R} = \{(0, 0), (0, 1), (0, 2), (1, 2)\} \ .$$

(A) { 1 point for each correct answer }
For each of the following statements determine whether it is true or false.
It is not necessary to justify your answer.

$\mathcal{R}$ is reflexive.	false
$\mathcal{R}$ is symmetric.	false
$\mathcal{R}$ is transitive.	true

(B) { 2 points }
Draw the directed graph for $\mathcal{R}$ **.**



Bonus. { 5 points } **Let** $\mathcal{S} = \{(0, 0), (0, 3), (1, 0), (1, 2), (2, 0), (3, 2)\}$ **be a binary relation defined on the set** $A = \{0, 1, 2, 3\}$**. Find** $\mathcal{S}^t$**, the transitive closure of** $\mathcal{S}$**.**
List the ordered pairs that have to be added to $\mathcal{S}$ **in order to receive** $\mathcal{S}^t$**.**

$$\mathcal{S}^t - \mathcal{S} = \{(0, 2), (1, 3), (2, 2), (2, 3), (3, 0), (3, 3)\} .$$