

Solutions for Quiz #11

November 13, 2003

1. Let $C = \{2, 3, 4, 5\}$ and $D = \{3, 4\}$. Define binary relation \mathcal{S} from C to D as follows

$$\text{for all } (x, y) \in C \times D, (x, y) \in \mathcal{S} \Leftrightarrow x \geq y.$$

(A) { 2 points } Write \mathcal{S} as a set of ordered pairs

$$\mathcal{S} = \{(3, 3), (4, 3), (4, 4), (5, 3), (5, 4)\}.$$

(B) { 1 point for each correct answer } For each of the following statements determine whether it is true or false. *It is not necessary to justify your answer.*

$$2 \mathcal{S} 4$$

false

$$4 \mathcal{S} 3$$

true

$$(2, 3) \in \mathcal{S}$$

false

2. The binary relation \mathcal{R} is defined on the set $A = \{0, 1, 2, 3\}$ with

$$\mathcal{R} = \{(0, 0), (0, 1), (0, 2), (1, 2)\}.$$

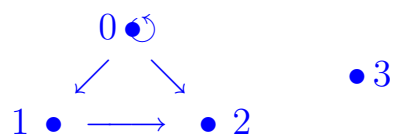
(A) { 1 point for each correct answer }
For each of the following statements determine whether it is true or false.
It is not necessary to justify your answer.

\mathcal{R} is reflexive. false

\mathcal{R} is symmetric. false

\mathcal{R} is transitive. true

(B) { 2 points }
Draw the directed graph for \mathcal{R} .



Bonus. { 5 points } Let $\mathcal{S} = \{(0, 0), (0, 3), (1, 0), (1, 2), (2, 0), (3, 2)\}$ be a binary relation defined on the set $A = \{0, 1, 2, 3\}$. Find \mathcal{S}^t , the transitive closure of \mathcal{S} .

List the ordered pairs that have to be added to \mathcal{S} in order to receive \mathcal{S}^t .

$$\mathcal{S}^t - \mathcal{S} = \{(0, 2), (1, 3), (2, 2), (2, 3), (3, 0), (3, 3)\}.$$