Practice Exam

1. Perform the mean and difference computation for a signal consisting of 8 samples and record the results in the first table. Then, round the numbers in the final row to the closest integer divisible by 10 and perform the inverse transform for the modified row in the second table. Use thresholding of all the differences whose absolute values are less than 10 and show the calculations of the inverse transform in the third table. Comment about the differences between these two approaches.

124	116	111	117	100	88	102	110

2. Design a lifting scheme based upon a prediction step using the approximation

$$s_j[2n+1] \approx \frac{1}{3}\left(2s_j[2n] + s_j[2n+2]\right).$$

Give a formula for the difference $d_{j-1}[n]$ and choose a formula for the update and $s_{j-1}[n]$ in such a way that the averages of the sequences s_{j-1} and s_j will be the same. What should you add to the scheme in order to preserve the energy? Assuming that the signal in the first row of the first table is 8-periodic, perform a wavelet decomposition of the signal using your filter and then reconstruct its high frequency content in the second table.

24	30	36	30	36	24	12	18

3. The following scheme is known as CDF(3,1)

$$s_{j-1}^{(1)}[n] = s_j[2n] - \frac{1}{3}s_j[2n-1]$$

$$d_{j-1}^{(1)}[n] = s_j[2n+1] - \frac{1}{8}\left(9s_{j-1}^{(1)}[n] + 3s_{j-1}^{(1)}[n+1]\right)$$

$$s_{j-1}^{(2)}[n] = s_{j-1}^{(1)}[n] + \frac{4}{9}d_{j-1}^{(1)}[n]$$

$$d_{j-1}[n] = \frac{\sqrt{2}}{3}d_{j-1}^{(1)}[n]$$

$$s_{j-1}[n] = \frac{3}{\sqrt{2}}s_{j-1}^{(2)}[n]$$

Do the following:

(a) Calculate the expressions describing directly $d_{j-1}[n]$ and $s_{j-1}[n]$ via $s_j[n]$;

(b) Check whether the scheme preserves the averages (with an appropriate normalization constant);

(c) Check whether the scheme preserves the first moment, namely that

$$\sum_n n s_{j-1}[n] = c\sum_n n s_j[n]$$

where c is the normalization constant;

(d) Check whether the scheme preserves the energy of the signal;

(e) Find the inverse transform in a form of a lifting scheme;

(f) Calculate the expressions for the inverse transform describing directly $s_j[n]$ via $d_{j-1}[n]$ and $s_{j-1}[n]$;

(g) Represent the lifting scheme in the z-transform and find the corresponding matrix $\mathbf{H}(z)$;

(h) Find the (z transforms of the) filters $H_0(z)$ and $H_1(z)$ representing the lifting scheme;

(i) Find the (z transforms of the) filters $G_0(z)$ and $G_1(z)$ representing the inverse transform;

(j) Check whether the above filters are orthogonal.

4. Let the four filters h_0, h_1, g_0, and g_1 satisfy the conditions

$$\sum_k g_0[k]h_0[2n-k] = \sum_k g_1[k]h_1[2n-k] = \delta[n]\ ,$$

$$\sum_k g_0[k]h_1[2n-k] = \sum_k g_1[k]h_0[2n-k] = 0$$

for all $n \in \mathbb{Z}$. Show that their z-transforms satisfy

$$G_0(z)H_0(z) + G_1(z)H_1(z) = 2 \quad \textbf{and} \quad G_0(z)H_1(z) + G_1(z)H_0(z) = 0$$