Quiz #8

SOLUTIONS

1. { 6 points } **Give a big-O estimate for each of these functions. For the function $g(n)$ in your estimate that $f(n)$ is $O(g)$, use a simple function g of the smallest order.**

(a) $f(n) = n\log(n^4+1) + \frac{n^2}{\log n}$

$$f(n) = O(n\log n) + O\left(\frac{n^2}{\log n}\right) = O\left(\frac{n^2}{\log n}\right) = O(n^2)$$

(b) $f(n) = (n\log n + n)^2 + \frac{2^n}{n!}$

$$f(n) = [O(n\log n)]^2 + O(1) = O\left(n^2(\log n)^2\right) = O(n^3)$$

2. { 4 points } **Show that**

$$\lceil 2x^2 - x\rceil = \Theta(x^2)$$

For $x > 1$ we have that

$$1 = \frac{x^2}{x^2} \le \frac{x^2 + x^2 - x}{x^2} = \frac{2x^2 - x}{x^2} \le \left|\frac{\lceil 2x^2 - x\rceil}{x^2}\right| \le \frac{2x^2}{x^2} = 2$$

Thus, $\left|\frac{\lceil 2x^2-x\rceil}{x^2}\right|$ is bounded by positive constants from below and above which proves that $\lceil 2x^2 - x\rceil = \Theta(x^2)$.