Quiz #7

SOLUTIONS

1. { 10 points } **Prove using mathematical induction that**

$$\sum_{j=1}^{n} j^2 \;=\; \frac{n(n+1)(2n+1)}{6}$$

whenever n **is a positive integer.**

Basis Step

For $n = 1$ **the statement is true since**

$$\sum_{j=1}^{n} j^2 = 1^2 = 1 = \frac{1(1+1)(2+1)}{6}$$

Inductive Step

Assume that The statement is true for $n = k$**, i.e.**

$$\sum_{j=1}^{k} j^2 \;=\; \frac{k(k+1)(2k+1)}{6}$$

Then for $n = k + 1$ **we have**

$$\begin{aligned} \sum_{j=1}^{k+1} j^2 &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = (k+1)\left(\frac{k(2k+1)}{6} + (k+1)\right) \\ &= (k+1)\left(\frac{2k^2+k \;+\; 6k+6}{6}\right) = (k+1)\left(\frac{(k+2)(2k+3)}{6}\right) \\ &= \frac{(k+1)\;\;((k+1)+1)\;\;(2(k+1)+1)}{6} \end{aligned}$$

which proves the statement for $n = k + 1$**.**