Quiz #6

SOLUTIONS

1. { 8 points } **Prove that there are no solutions in positive integers x and y to the equation**

$$x^3 + y^3 = 19$$

For positive integers x and y we have that x^3 and y^3 are positive, so $x^3 < 19$ and $y^3 < 19$. Thus, $x = 1, 2$ and $y = 1, 2$ are the only possible values since $\sqrt[3]{19} < 3$. We check all the possibilities:

$$x = 1,\ y = 1\ : \qquad x^3 + y^3 = 1 + 1 = 2 \neq 19$$

$$x = 1,\ y = 2\ : \qquad x^3 + y^3 = 1 + 8 = 9 \neq 19$$

$$x = 2,\ y = 1\ : \qquad x^3 + y^3 = 8 + 1 = 9 \neq 19$$

$$x = 2,\ y = 2\ : \qquad x^3 + y^3 = 8 + 8 = 16 \neq 19$$

Therefore, there are no solutions to the equation in positive integers.

Note that we can skip the case $x = 2,\ y = 1$ due to its symmetry with $x = 1,\ y = 2$.

2. { 2 points } **Prove or disprove that there are no solutions in integers x and y to the equation**

$$x^3 + y^3 = 19$$

For $x = -2$ and $y = 3$ we have that $(-2)^3 + 3^3 = -8 + 27 = 19$. This example disproves the proposition that there are no solutions to the equation in (arbitrary) integers.