Quiz #5

SOLUTIONS

1. { 10 points } **Let** $f : \mathbb{R} \to \mathbb{R}$ **and let** $f(x) < 0$ **for all** $x \in \mathbb{R}$**. Prove that** $f(x)$ **is strictly increasing if and only if the function** $g(x) = \frac{1}{f(x)}$ **is strictly decreasing.**

By the definition $f(x)$ **is strictly increasing, if**

$$\forall x \forall y \, (x < y \to f(x) < f(y)) \, .$$

Dividing both parts of the inequality $f(x) < f(y)$ **by** $f(x)f(y) > 0$ **gives** $\frac{1}{f(y)} < \frac{1}{f(x)}$ **which is equivalent to** $g(y) < g(x)$**. Thus,**

$$\forall x \forall y \, (x < y \to g(x) > g(y))$$

which means that $g(x)$ **is strictly decreasing.**

To prove the inverse direction, let $g(x) = \frac{1}{f(x)}$ **be strictly decreasing. Then,**

$$\forall x \forall y \, (x < y \to g(x) > g(y)) \, .$$

Using that $g(x) > g(y)$ **is equivalent to** $f(x) < f(y)$ *(which we proved above,)* **we receive**

$$\forall x \forall y \, (x < y \to f(x) < f(y))$$

which means that $f(x)$ **is strictly increasing.**