Quiz #3

SOLUTIONS

1. { 10 points } **Prove that if x is irrational, then $\frac{1}{x}$ is irrational.**

Note first that $x \neq 0$ since it is not rational but 0 is. Thus, $\frac{1}{x}$ is well defined. We also have that $\frac{1}{x} \neq 0$ since its inverse x exists.
The point of the above remark is to justify that x and $\frac{1}{x}$ are well defined real numbers and we can formulate the statements below. In general, it is a step we shall omit most of the times.

We use indirect proof. The implication is equivalent to:
If $\frac{1}{x} \neq 0$ is rational, then x is rational.
To prove the above statement, we use that for the rational number $\frac{1}{x}$ there exist two integers p and $q \neq 0$ such that $\frac{1}{x} = \frac{p}{q}$. Since $\frac{1}{x} \neq 0$, we have that $p = q\frac{1}{x} \neq 0$ and therefore $x = \frac{q}{p}$, where q and $p \neq 0$ are integers. Thus, x is rational.