

Take-Home Quiz # 21

Write your name and code on the back. **Return by 9:05 AM on November 29, 2006.**

1. { 20 points } For the version of the game of **nim** described in the textbook, prove that the first player wins, if both players follow an optimal strategy, unless the initial position can be described by one of the following two cases:

(A) All the piles have exactly one stone and the number of piles is odd.

(B) There are at least two piles with more than one stone and the quantities of stones in the piles represented as binary numbers $(\sum_{k=0}^n b_k 2^k)$ have the following property: for every $k = 0, 1, \dots$ the number of ones in the k -th position (corresponding to the coefficients b_k in front of 2^k) for all the piles is even.