

Quiz #17

SOLUTIONS

1. { 10 points } Find the number of paths of length 3 between

(a) two different vertices in K_4 .

Let the vertices of the graph are v_1 , v_2 , v_3 , and v_4 . The possible paths between two of them, say v_1 and v_4 are:

$$\begin{aligned} &v_1 \mapsto v_2 \mapsto v_1 \mapsto v_4 && v_1 \mapsto v_2 \mapsto v_3 \mapsto v_4 \\ &v_1 \mapsto v_3 \mapsto v_1 \mapsto v_4 && v_1 \mapsto v_3 \mapsto v_2 \mapsto v_4 \\ &v_1 \mapsto v_4 \mapsto v_1 \mapsto v_4 && v_1 \mapsto v_4 \mapsto v_2 \mapsto v_4 && v_1 \mapsto v_4 \mapsto v_3 \mapsto v_4 \end{aligned}$$

The problem can be solved also by using the adjacency matrix:

$$A^3 = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}^3 = \begin{pmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 7 & 7 & 7 \\ 7 & 6 & 7 & 7 \\ 7 & 7 & 6 & 7 \\ 7 & 7 & 7 & 6 \end{pmatrix}$$

(b) two adjacent vertices in $K_{3,3}$.

Let u_1 and u_4 be two fixed adjacent vertices from the graph. Assume that the path is

$$u_1 \mapsto u_2 \mapsto u_3 \mapsto u_4.$$

Then, the vertex u_2 could be any of the three vertices adjacent to u_1 and nonadjacent to u_4 , while (regardless of the choice of u_2), u_3 could be any of the three vertices adjacent to u_4 and nonadjacent to u_1 . That makes a total of $3 \cdot 3 = 9$ possible paths.

Note that a solution using the adjacency matrix is also possible but it requires too much space to be included here in full.

$$A^3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}^3 = \begin{pmatrix} 0 & 0 & 0 & 9 & 9 & 9 \\ 0 & 0 & 0 & 9 & 9 & 9 \\ 0 & 0 & 0 & 9 & 9 & 9 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \end{pmatrix}$$