

Quiz #14

SOLUTIONS

1. { 10 points } Solve the recurrence relation together with the initial condition given.

$$a_n = -4a_{n-1} + 5a_{n-2} \quad \text{for } n \geq 2, \quad a_0 = 2, \quad a_1 = 8$$

The characteristic equation for this relation is

$$r^2 - (-4)r - 5 = r^2 + 4r - 5 = 0$$

which has roots $r_1 = 1$ and $r_2 = -5$.

Thus, the general solution for the recurrence relation is

$$a_n = \alpha_1 1^n + \alpha_2 (-5)^n .$$

From the initial conditions, it follows that

$$a_0 = 2 = \alpha_1 + \alpha_2$$

$$a_1 = 8 = \alpha_1 - 5\alpha_2$$

Solving these two equations gives $\alpha_1 = 3$ and $\alpha_2 = -1$.

Hence, the solution is

$$a_n = 3 - (-5)^n$$