

Name : _____

Exam 2 March 22, 2005

DIRECTIONS: Clearly mark all your answers. You may use calculators to check your work. **Show all work!** There is a total of 100 possible points on this exam. **Do the Bonus problems last!**

1. (10 points) Solve the LP problem. If no optimal solution exists, indicate whether the feasible region is empty or the objective function is unbounded.

$$\begin{aligned} &\text{Maximize } p = x + 2y \\ &\text{subject to } x + 2y \geq 6 \\ &\quad 2x + y \geq 6 \\ &\quad x \geq 0, y \geq 0 \end{aligned}$$

2. (10 points) Prove one of DeMorgan's Laws; that is prove that

$$\sim (p \vee q) \equiv (\sim p) \wedge (\sim q).$$

Show all of your work in a truth table. For extra credit state the other law.

3. (12 points) Prove whether or not the statement $p \leftrightarrow [(\sim p) \wedge (q)] \equiv ((\sim p) \leftrightarrow [p \vee (\sim q)])$ is tautology. Be sure to state your answer in a sentence following your work.

4. (10 points) Suppose that you and a friend construct a modified version of paper, rock, scissors with the payoff matrix:

$$\begin{bmatrix} -3 & 1 & 0 \\ 2 & -1 & -2 \\ 5 & -6 & -1 \end{bmatrix}$$

Label player A (the row player), player B (the column player), and the rows.

- Find the optimal strategy for each player.
- Is there a saddle point?
- Which player would you rather be player A or B? Explain your answer for credit.

5. (12 points) Calculate the following:

(a) Suppose $A = \{1, 3, 4, 5, 7, 217\}$ and $B = \{4, 5, 119\}$. Find $A \cup B$ and $A \cap B$.

(b) Suppose that $|A| = 413$, $|B| = 21$, and $|A \cap B| = 11$. Calculate $|A \cup B|$.

(c) Suppose C is the set of outcomes of rolling two dice such that the numbers add to 8. Find $|C|$.

6. (10 points) How many three-letter (ordered) sequences are possible that use the letters a, b, c, d, g at most once.

7. (24 points) Suppose a bag contains 3 red, 2 green, 4 yellow, and 1 blue marble.

- (a) How many possible sets of 4 marbles are there?
- (b) How many possible sets of 4 include all the red marbles?
- (c) How many possible sets of 4 include one of each color marble?
- (d) How many possible sets of 4 including all the red and green marbles are there?

8. (12 points) A poker hand consists of 5 cards from a standard deck of 52 cards. Find the number of different poker hands with two-of-a-kind (i.e. two of one denomination and three of different denominations).

Bonus:

- 1. (1 point) Name one mathematician whom we talked about between the time after the first exam and now.
- 2. (5 point) Calculate the odds of winning the lottery, if each ticket contains 5 distinct numbers ranging from 1-50. For partial credit, you could determine how many possible tickets there are. (Note that order does not matter.)