1. What is the area of the trapezoid below?

![Trapezoid Diagram]

(a) $\frac{2 + \sqrt{3}}{4}$  
(b) 1  
(c) 2  
(d) $\frac{5 + 2\sqrt{3}}{2}$  
(e) $\frac{7 - 2\sqrt{3}}{2}$

2. The value of $\frac{x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7}{x^{-3} + x^{-4} + x^{-5} + x^{-6} + x^{-7} + x^{-8} + x^{-9}}$ is

(a) $x^{20}$  
(b) $x^{16}$  
(c) $x^{10}$  
(d) $x^{-7}$  
(e) $x^{-2}$

3. As you may recall, the sum of the interior angles of a triangle is 180°. How many sides must a polygon have if the sum of the interior angles is 2520°?

(a) 12  
(b) 14  
(c) 16  
(d) 28  
(e) 42

4. An item for sale is marked down 20%. By what percent must it then be marked up in order to return to the original selling price?

(a) 20  
(b) 22  
(c) 23  
(d) 24  
(e) 25

5. Suppose that the number of elements in the set $S$ is 105 and that $S$ is split into $n$ subsets of $11m + 2$ elements each. If $m$ is an integer, then $m$ is

(a) 2  
(b) 3  
(c) 4  
(d) 5  
(e) 6

6. Suppose five ordinary dice are rolled. What is the probability that at least one 6 appears?

(a) $1 - \left(\frac{5}{6}\right)^5$  
(b) $\left(\frac{1}{6}\right)^5$  
(c) $\left(\frac{5}{6}\right)^5$  
(d) $\frac{1}{6}$  
(e) $\frac{5}{6}$
7. Suppose \( \cos(\theta) + \cos(2\theta) = 0 \) and \( 0 \leq \theta \leq \pi/2 \). Then \( \cos(\theta) = \)

(a) 0  
(b) 1  
(c) 1/2  
(d) \( \sqrt{3}/2 \)  
(e) \( \sqrt{2}/2 \)

8. Suppose that 1 and 2 are roots of \( x^3 + ax^2 + bx + c = 0 \) and that \( a + b = -15 \). Then \( a = \)

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) 5

9. In a plane, points \( A \) and \( B \) are on one side of line \( \ell \). If \( A \) and \( B \) are both 3 centimeters from \( \ell \) and are 4 centimeters from each other, find the radius of the circle that is tangent to \( \ell \) and goes through \( A \) and \( B \).

(a) 13/6 cm  
(b) 7/3 cm  
(c) 15/6 cm  
(d) 8/3 cm  
(e) 17/6 cm

10. \( \sqrt{3} + \sqrt{2} + \sqrt{3 - \sqrt{2}} = \)

(a) \( \sqrt{6} \)  
(b) \( 2\sqrt{3} \)  
(c) \( \sqrt{3} + \sqrt{3 - \sqrt{2}} \)  
(d) 3.5  
(e) \( \sqrt{6 + \sqrt{28}} \)

11. The largest solution of \( x^6 - x^5 - x^4 - x^3 - x^2 - x - 1 = 0 \) is in which of the following intervals?

(a) \([0, 1)\)  
(b) \([1, 2)\)  
(c) \([2, 3)\)  
(d) \([3, 4)\)  
(e) \([4, \infty)\)

12. For how many positive integer values of \( n \) is \( 1! + 2! + 3! + \cdots + n! \) a perfect square?

(a) none  
(b) 1  
(c) 2  
(d) 3  
(e) infinitely many
13. Suppose that the array of squares below is filled with positive integers in such a way that the product of each row, column, and each of the two main diagonals is the same. What must be the value in the lower right-hand square?

\[
\begin{array}{cc}
36 & 2 \\
6 & \\
??? & \\
\end{array}
\]

(a) 8  (b) 12  (c) 16  (d) 24  (e) 36

14. Both solution X and solution Y contain alcohol and water. In solution X, the ratio of alcohol to water is 3:2. When equal amounts of solution X and solution Y are mixed, the ratio of alcohol to water is 3:4. What is the ratio of alcohol to water in solution Y?

(a) 1:1  (b) 9:26  (c) 10:25  (d) 10:24  (e) none of these

15. It is O’s move in the Tic-Tac-Toe game below. Which of the moves 1, 2, 3, 4, or 5 below is the best move for O?

\[
\begin{array}{ccc}
X & O & 1 \\
2 & 3 & 4 \\
5 & X & \\
\end{array}
\]

(a) 1  (b) 2  (c) 3  (d) 4  (e) 5

16. The largest integer k such that \(5^k\) divides \(3(10!) + 12(5!) + 4(7!)\) is

(a) 1  (b) 2  (c) 3  (d) 4  (e) 5
17. Suppose that \( t \) is an integer, \( 1 \leq t \leq 100 \), and that the equation
\[ x^2 + tx + \left( \frac{t^2 - t}{4} \right) = 0 \] has an integral root. Then the number of possible values of \( t \) is:

(a) 6  (b) 7  (c) 8  (d) 9  (e) 10

18. \( \tan (2 \arcsin (x)) = \)

(a) \( \frac{2\sqrt{1-x^2}}{1-2x^2} \)  (b) \( \frac{2x\sqrt{1-x^2}}{1-2x^2} \)  (c) \( \frac{2x\sqrt{1+x^2}}{1+2x^2} \)

(d) \( \frac{x\sqrt{1-x^2}}{1-2x^2} \)  (e) \( \frac{2\sqrt{1+x^2}}{1+2x^2} \)

19. \( (\sin 20^\circ)(\tan 10^\circ + \cot 10^\circ) \) is equal to

(a) \( \frac{3}{2} \)  (b) 3  (c) \( \frac{1}{3} \)  (d) 2  (e) \( \frac{1}{2} \)

20. Circle \( C \) is inscribed in square \( PQRS \), and the length of segment \( PQ \) is 1. Inside square \( PQRS \), circle \( D \) is tangent to circle \( C \), segment \( PQ \) and segment \( QR \). Find the radius of circle \( D \).

\[
\text{(a) } \frac{3 - 2\sqrt{2}}{2} \quad \text{(b) } \frac{\sqrt{2} - 1}{1 + 2\sqrt{2}} \quad \text{(c) } \frac{\sqrt{2} - 1}{2} \quad \text{(d) } \frac{\sqrt{2} + 1}{8} \quad \text{(e) } 2 - \sqrt{2}
\]

21. Which of the following is equal to \( \sum_{k=1}^{99} \log_{10} \left( \frac{k+1}{k} \right) \) ?

(a) 0  (b) 1  (c) 2  (d) \( \log_{10} 99 \)  (e) \( \log_{10}(100/99) \)

22. What is the base of the numeral system in which \( 1/5 = .333\ldots \)?

(a) 7  (b) 9  (c) 11  (d) 14  (e) 16
23. If $S$ is the set of all points $z$ in the complex plane such that $(3 + 4i)z$ is a real number, then $S$ is a

(a) line    (b) triangle    (c) circle    (d) hyperbola    (e) parabola

24. How many integers $x$ are there such that $1 \leq x \leq 100$, and $x^3 + 4x + 2$ is divisible by 7?

(a) 26    (b) 27    (c) 28    (d) 29    (e) 30

25. Moving along the line segments of this grid in the directions North and East only, how many different paths are there from $A$ to $B$? Notice that one segment is missing.

\[ \text{Grid Diagram} \]

(a) 320    (b) 342    (c) 356    (d) 382    (e) 390

26. If $16^{101} + 8^{101} + 4^{101} + 2^{101} + 1$ is divided by $2^{100} + 1$, then the remainder is

(a) 0    (b) 2    (c) 4    (d) 11    (e) 101

27. Suppose that the straight line $L$ meets the curve $y = 3x^3 - 15x^2 + 7x - 8$ in three points $(x_1, y_1), (x_2, y_2),$ and $(x_3, y_3)$. Then $x_1 + x_2 + x_3 =$

(a) 3    (b) 4    (c) 5    (d) 6    (e) 7
28. Suppose that $S$ is a subset of the integers that contains at least two elements and is such that:

For all integers $x$ and $y$, if $x \in S$ and $y \not\in S$, then $x + y \not\in S$.

How many of the following statements are true about $S$?

(i) $0 \in S$.
(ii) If $x \in S$, then $-x \in S$.
(iii) If $x \in S$ and $y \in S$, then $x + y \in S$.
(iv) $S$ must be infinite.

(a) none    (b) exactly 1    (c) exactly 2    (d) exactly 3    (e) all

29. Find the maximum diameter of a circle that lies completely on or above the $x$ axis, passes through the origin, and intersects the parabola $y = x^2$ only at the origin.

(a) $1/2$    (b) $\sqrt{2}/2$    (c) $\sqrt{3}/2$    (d) 1    (e) $\sqrt{2}$

30. Suppose that $x_1, x_2, \ldots, x_{40}$ and $y_1, y_2, \ldots, y_{40}$ are two permutations of the integers $1, 2, \ldots, 40$. Then the value of $\sum_{k=1}^{40} (2x_k + y_k)$ is

(a) 2360    (b) 2420    (c) 2460    (d) 2520

(e) Cannot be determined from this information