When describing some biological flows, small length and time scales allow inertia to be neglected in mathematical models, and the fluid dynamics may be described by the linear Stokes equation. However, when the flow is coupled to passive or actuated elastic structures, nonlinear behavior can occur. We will discuss some examples of these complex systems in the context of cilia, flagella and viscoelastic networks at the micro scale.