**February/March 2012 Solutions**

**Sum of Logs**

For any real number $x$, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to $x$. Without relying on technology, compute the sum

$$[\log_3 1] + [\log_3 2] + \ldots + [\log_3 1000].$$

**Solution**

Observe for $1 \leq n < 3$, that $[\log_3 n] = 0$. Similarly for $3 \leq n < 3^2 = 9$, we have $[\log_3 n] = 1$, and for $3^2 \leq n < 3^3$, the quantity $[\log_3 n] = 2$. In general for $3^k \leq n < 3^{k+1}$, $[\log_3 n] = k$. Note that $3^6 < 1000 < 3^7$, so the sum is equal to

$$1 \cdot (9 - 3) + 2 \cdot (27 - 9) + 3 \cdot (81 - 27) + 4 \cdot (243 - 81) + 5 \cdot (729 - 81) + 6 \cdot (1000 - 729) + 1 = 4914$$

**Correct Solutions**

(1) Daniel Grier
(2) Nolan Miller
(3) Stefan Singer
(4) Daniel Wallis
(5) Elizabeth Minten
(6) David Hughey
(7) Robert Moyer
(8) Ryan Benitez
(9) Runyu Bi