

Syllabus for the admission to candidacy examination in algebra 700-701

Linear Algebra

Vector spaces, subspaces, spanning sets, linear independence, bases, dimension, direct sums and products, the algebra of linear transformations, linear functionals, dual spaces and dual bases, rank and nullity, determinants, eigenvalues and eigenvectors, diagonalizable matrices, minimal and characteristic polynomials, the Cayley-Hamilton theorem, rational and Jordan canonical forms, Hermitian inner products on complex vector space, orthonormal basis, unitary diagonalization of normal operators and matrices.

Group Theory

Groups, subgroups, cosets, homomorphisms, normal subgroups, quotient groups, Lagrange Theorem, direct sums and products, cyclic groups, symmetric groups, alternating groups, dihedral groups, Sylow subgroups and the Sylow theorems, free groups, presenting groups by generators and relations, the structure of finitely generated Abelian groups, simple groups, solvable groups.

Ring and module theory

Rings, homomorphisms, ideals, modules, polynomial rings, quotient rings, prime and irreducible elements, UFD's, Euclidian domains, principal ideal domains, the structure of finitely generated modules over principal ideal domains, prime ideals, maximal ideals, the Chinese Remainder Theorem, the field of fractions of a domain, the Nullstellensatz.

Syllabus for Math 700

Vector spaces, subspaces, spanning sets, linear independence, bases, dimension, direct sums and products, the algebra of linear transformations, linear functionals, dual spaces and dual bases, rank and nullity, determinants, eigenvalues and eigenvectors, diagonalizable matrices, minimal and characteristic polynomials, the Cayley-Hamilton theorem, similarity of matrices, Hermitian inner products on complex vector space, orthonormal basis, unitary diagonalization of normal operators and matrices.

Syllabus for Math 701

Group theory Groups, subgroups, cosets, homomorphisms, normal subgroups, quotient groups, Lagrange Theorem, direct sums and products, cyclic groups, symmetric groups, alternating groups, dihedral groups, Sylow subgroups and the Sylow theorems, free groups, presenting groups by generators and relations, the structure of finitely generated Abelian groups, simple groups.

Optional: solvable groups, the Jordan-Holder theorem.

Ring and module theory Rings, homomorphisms, ideals, modules, polynomial rings, quotient rings, prime and irreducible elements, UFD's, Euclidian domains, principal ideal domains, the structure of finitely generated modules over principal ideal domains (including the rational and Jordan canonical forms for matrices deduced as corollaries), prime ideals, maximal ideals, the Chinese Remainder Theorem, the field of fractions of a domain.

Optional: localization, the Nullstellensatz.